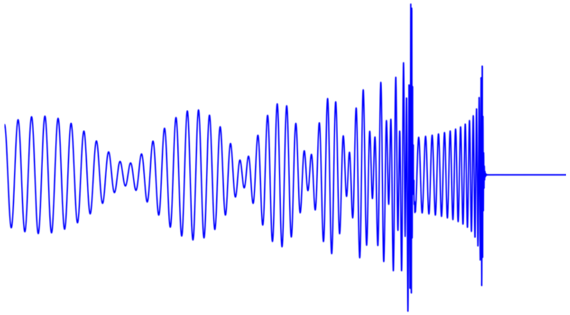

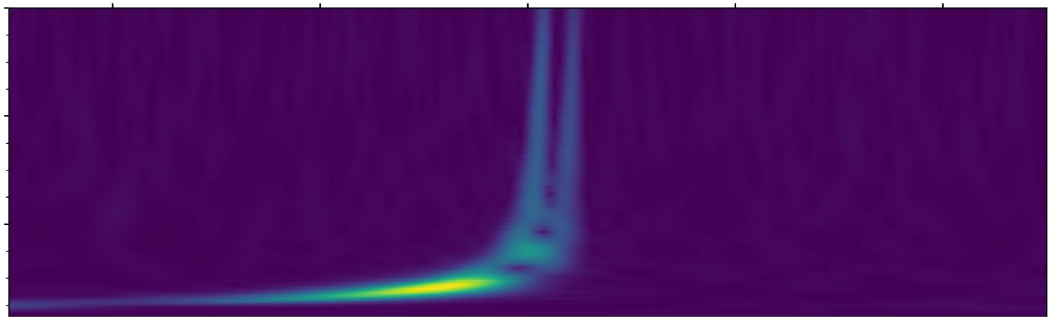

Interference signatures from gravitational lensing on gravitational waves

«Lensing and wave optics in strong gravity» Workshop
December 2024

Helena Ubach
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Institute of Cosmos Sciences (ICCUB), University of Barcelona
Virgo group at ICCUB

Outline

- Introduction
  - Gravitational waves
  - Gravitational lensing

- Wave effects on gravitational lensing of gravitational waves

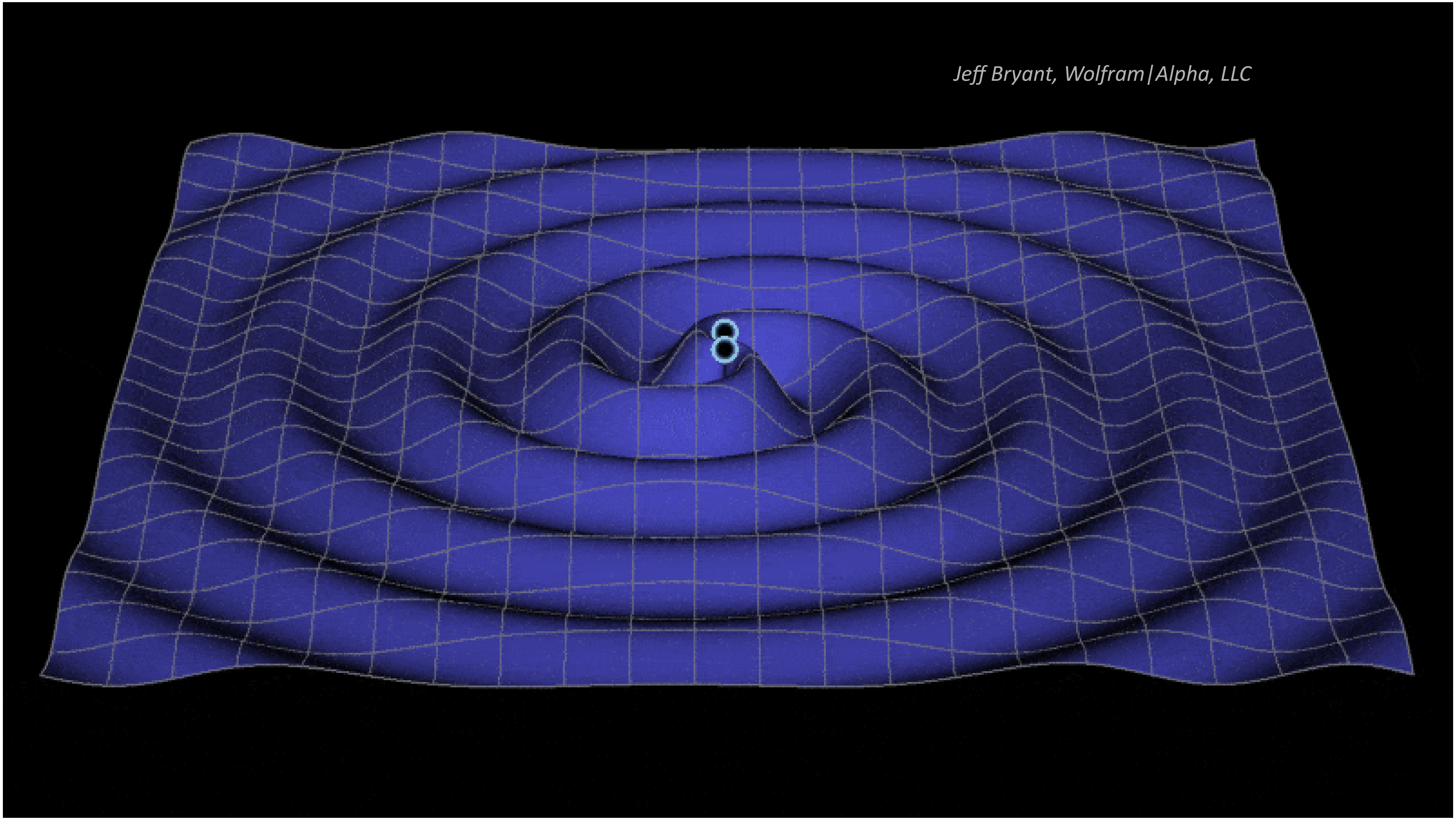
  1. Why do they appear?
  2. When are they significant?
  3. How do they look like? [e.g. source = compact binary merger]
  + Sonification
  4. *Are they detectable?*

- Conclusion
- Current work

Work in collaboration with Oleg Bulashenko, Ruxandra Bondarescu, Andrew Lundgren, ♪ Jordi Espuny

Current work with Mark Gieles and Jordi Miralda-Escudé

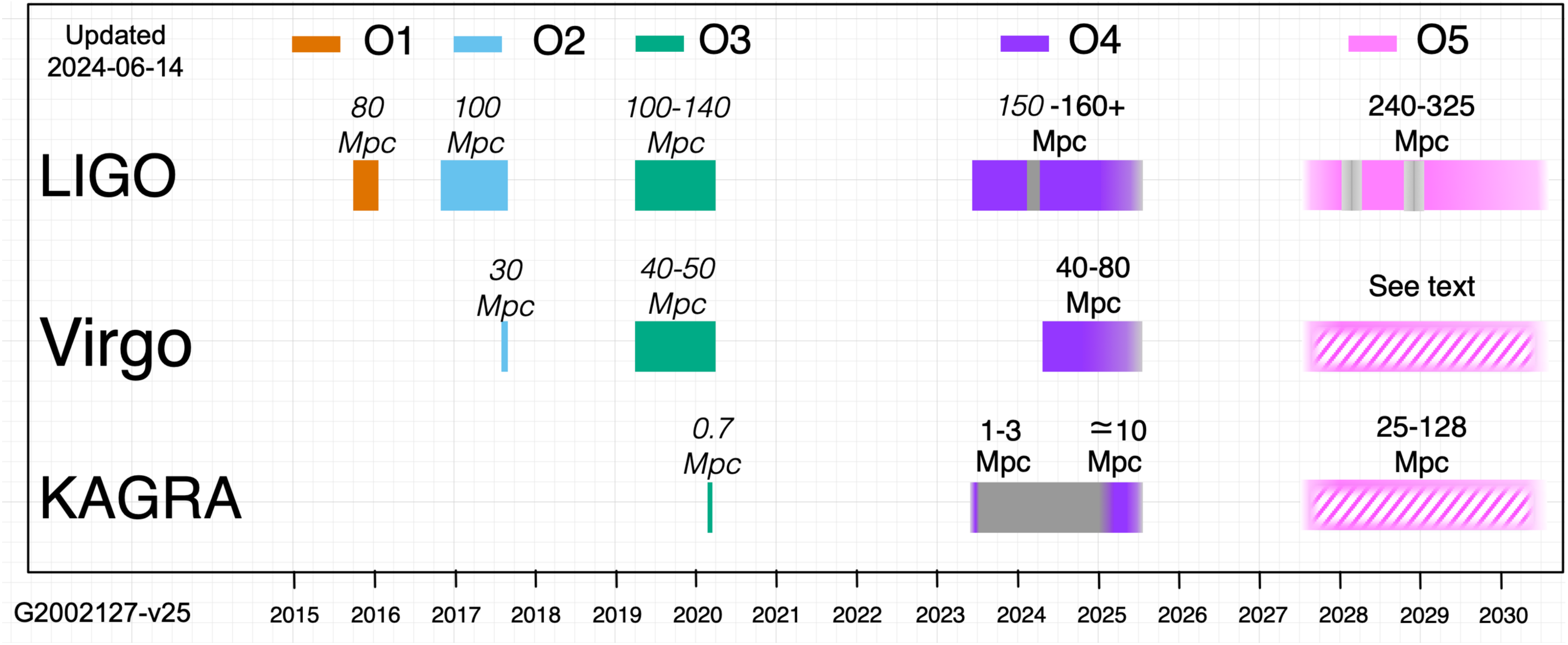
Introduction - Gravitational waves



Jeff Bryant, Wolfram|Alpha, LLC

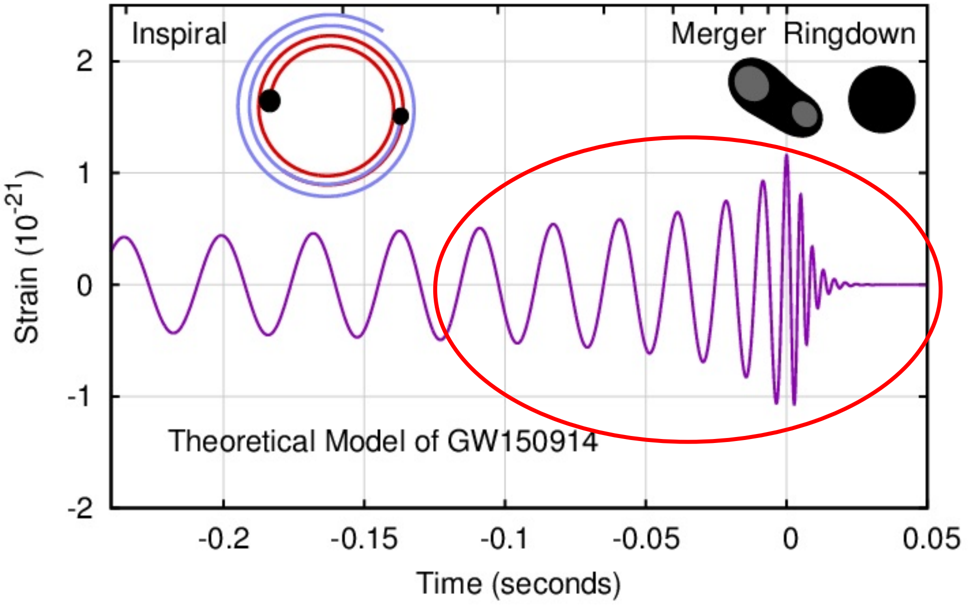
Gravitational waves: detections

https://observing.docs.ligo.org/plan/



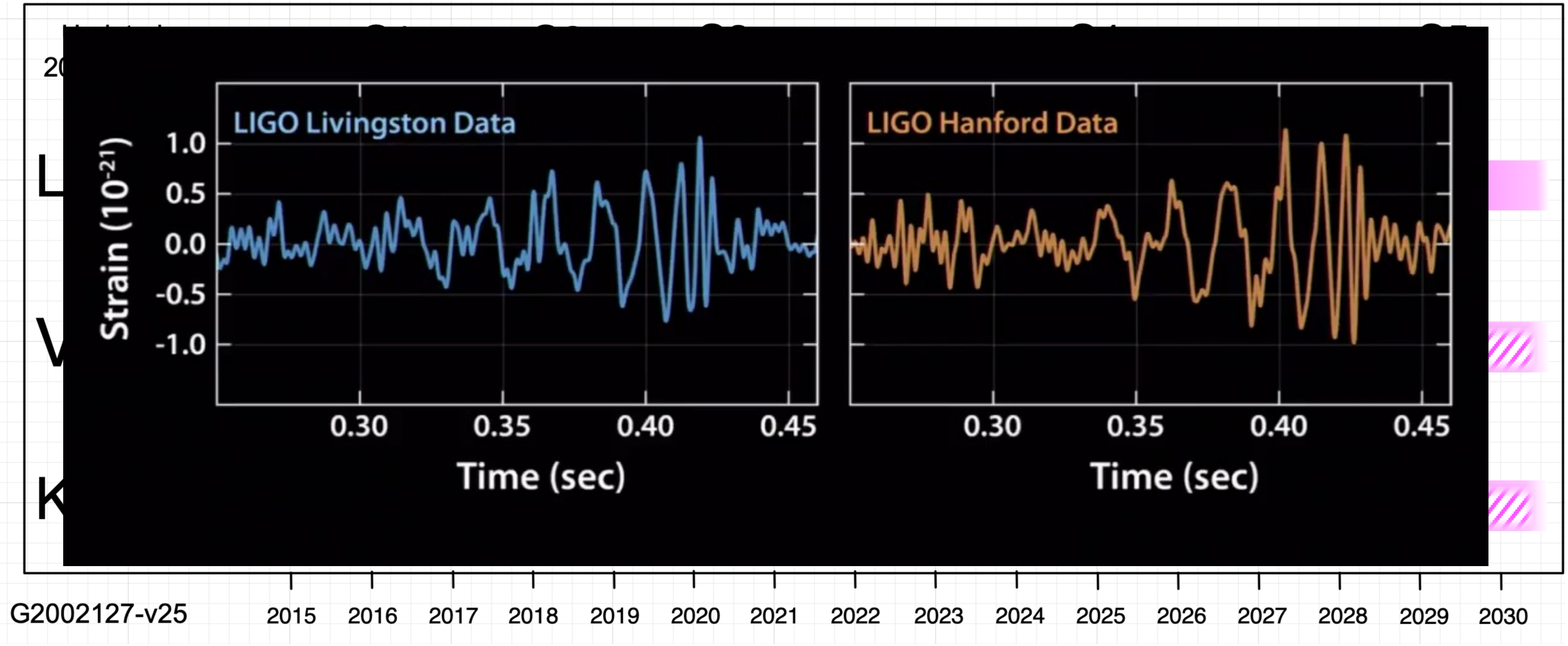
Merger of
2 compact
objects
(neutron stars,
black holes)

Rochester Institute of Technology



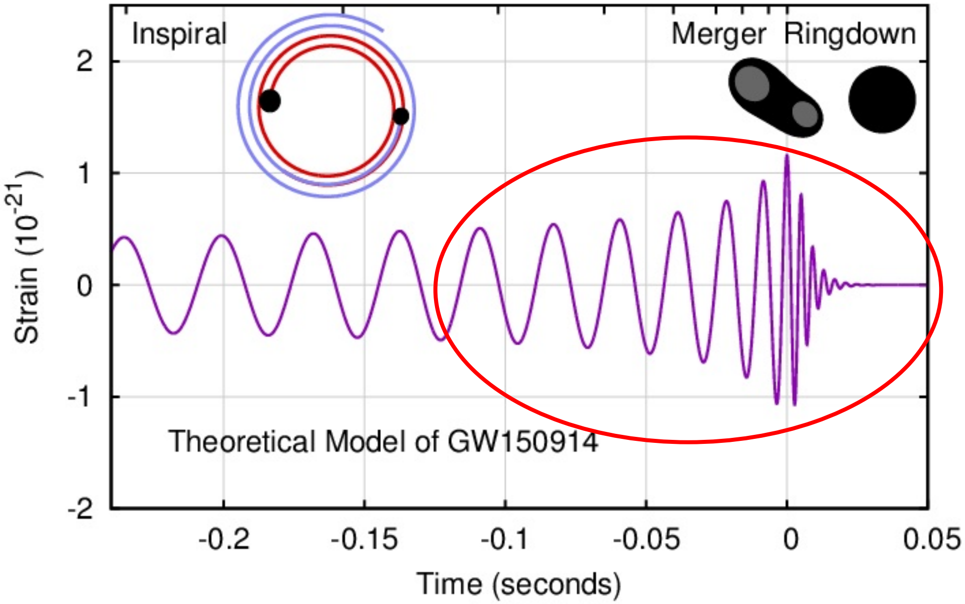
Last orbits

Gravitational waves: detections



https://observing.docs.ligo.org/plan/

Merger of
2 compact
objects
(neutron stars,
black holes)



Last
orbits

Rochester Institute of Technology

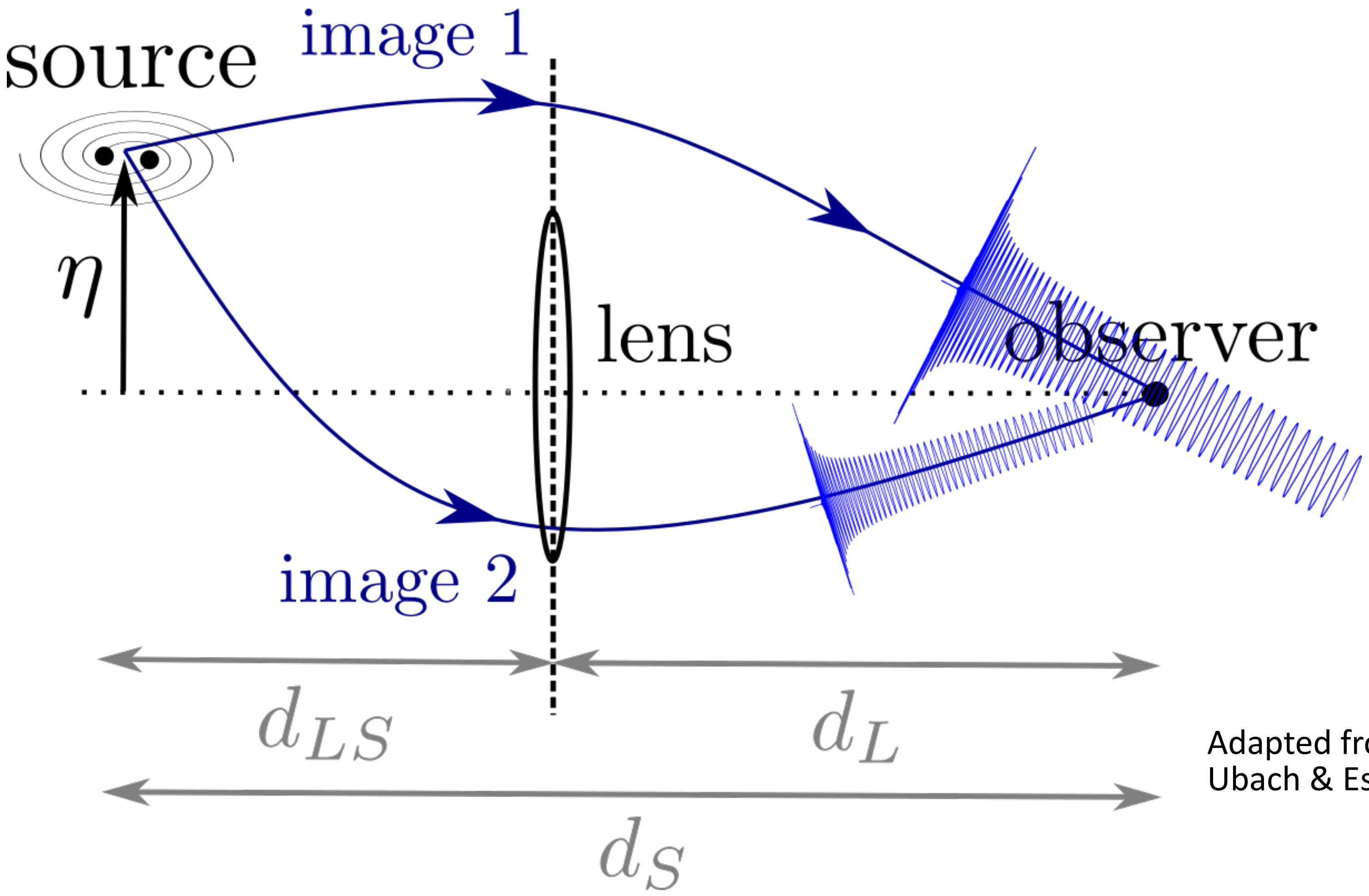
Gravitational waves: some characteristics

- Similarly to light, gravitational waves can also be gravitationally lensed
  Assumption: treating them as a scalar field

- Practically not absorbed, dispersed: they travel undistorted, except by gravity (lensing effect) and redshift

- Emitted in a coherent way → wave effects could be observed

Introduction - Gravitational lensing (of gravitational waves)



Adapted from
Ubach & Espuny 2024

***Geometric Optics**
***Point mass lens model (2 images)**

Introduction - Gravitational lensing (of gravitational waves)

Gravitational wave images ↔ light images?

- **Transient**: we see each signal for a very short time
- Not visible as separated in the sky → **images separated in time**

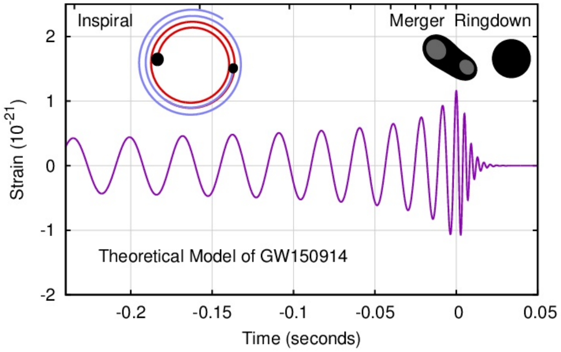
Separation in time: better resolvability of the images compared to angular resolution in the case of light

Gravitational waves:

- Lens mass for strong lensing: $M_L \gtrsim 10^4 \, M_\odot$

Light:

- Lens mass for strong lensing: $M_L \gtrsim 10^6 \, M_\odot$



Wave effects

1. Why do wave effects appear?

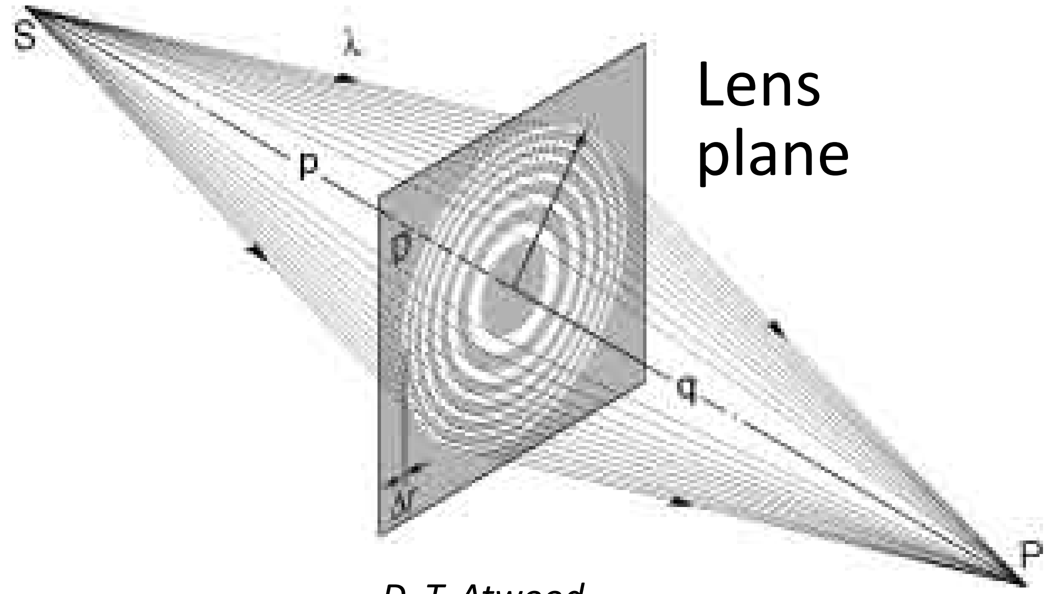
Wave effects appear as:

- Diffraction (around the lens)
- Interference (between emerging images)

Gravitational waves have:

- Coherent emission (Thorne, 1994)
- Long wavelength: $\lambda \sim R_{\mathrm{S}}$ $(y \sim 1)$ → wave effects more common

1. Why do wave effects appear?



Source

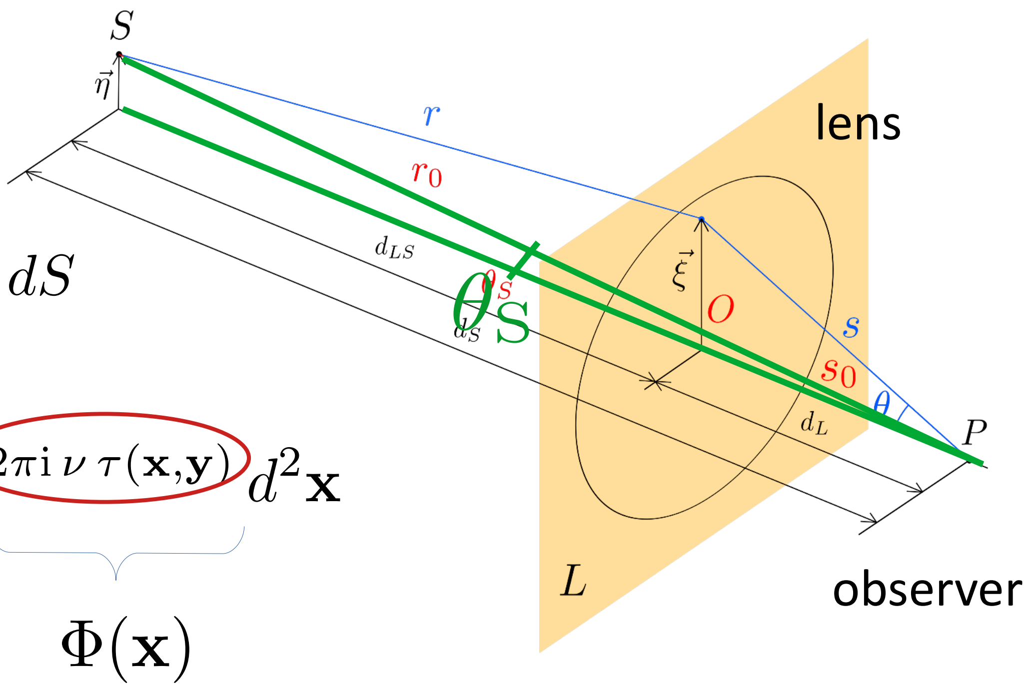
Lens plane

Observer

D. T. Atwood

1. Why do wave effects appear?

Scalar field at the observer:



$$\widetilde{\varphi}(P) = \frac{A}{\mathrm{i}\lambda} \iint \frac{1}{rs} \, e^{\mathrm{i}[k(r+s)+\psi_L]} \, dS$$

$$= -\mathrm{i}\nu \frac{A}{d_S} e^{\mathrm{i}k(r_0+s_0)} \iint \underbrace{e^{2\pi \mathrm{i}\,\nu\,\tau(\mathbf{x},\mathbf{y})}}_{\Phi(\mathbf{x})} d^2\mathbf{x}$$

Parameters: $\tau(y), \nu$:

$$\tau_{21} \equiv \tau(x_2, y) - \tau(x_1, y)$$

$$\tau(\mathbf{x},\mathbf{y}) = \frac{1}{2}(\mathbf{x}-\mathbf{y})^2 - \psi(\mathbf{x}) + \psi_0 \longrightarrow \tau_{21}(y) = 2y + \frac{1}{12}y^3 + O(y^5)$$

Point mass lens model

Schwarzschild radius of the lens $R_S = \frac{2GM}{c^2}$

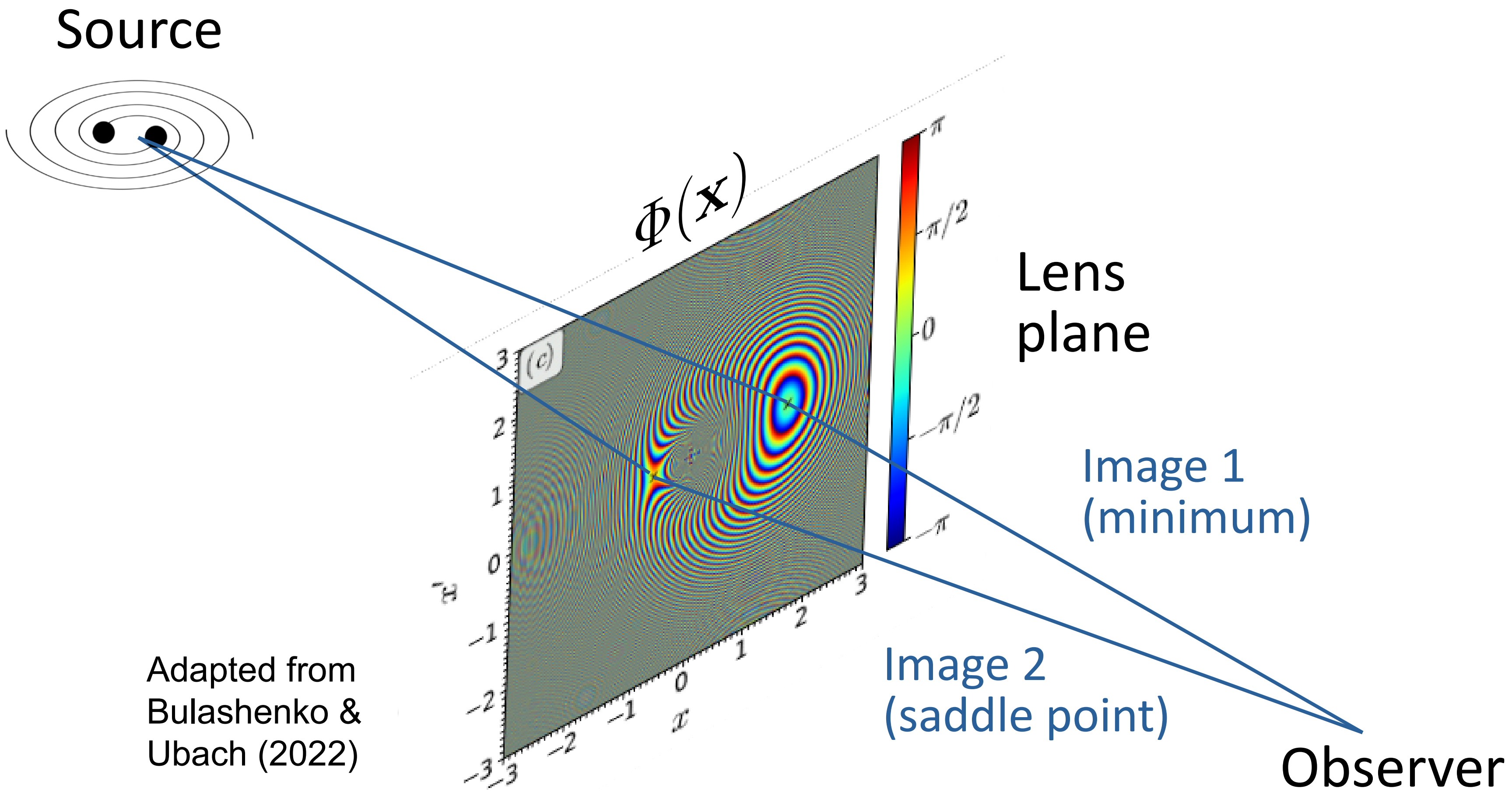
$$\nu = \frac{2R_{\rm S}}{\lambda}$$

Wavelength of gravitational waves

$$\nu = \frac{2R_{\rm S}}{c} f$$

$$y = \frac{\theta_{\rm S}}{\theta_{\rm E}}$$

1. Why do wave effects appear?



Adapted from Bulashenko & Ubach (2022)

$$\widetilde{\varphi}(P) \propto \sum_j |\mu_j|^{1/2} e^{i(2\pi\nu\,\tau(\mathbf{x}_j,\mathbf{y}) - n_j\pi/2)}$$

$$\nu = 10$$

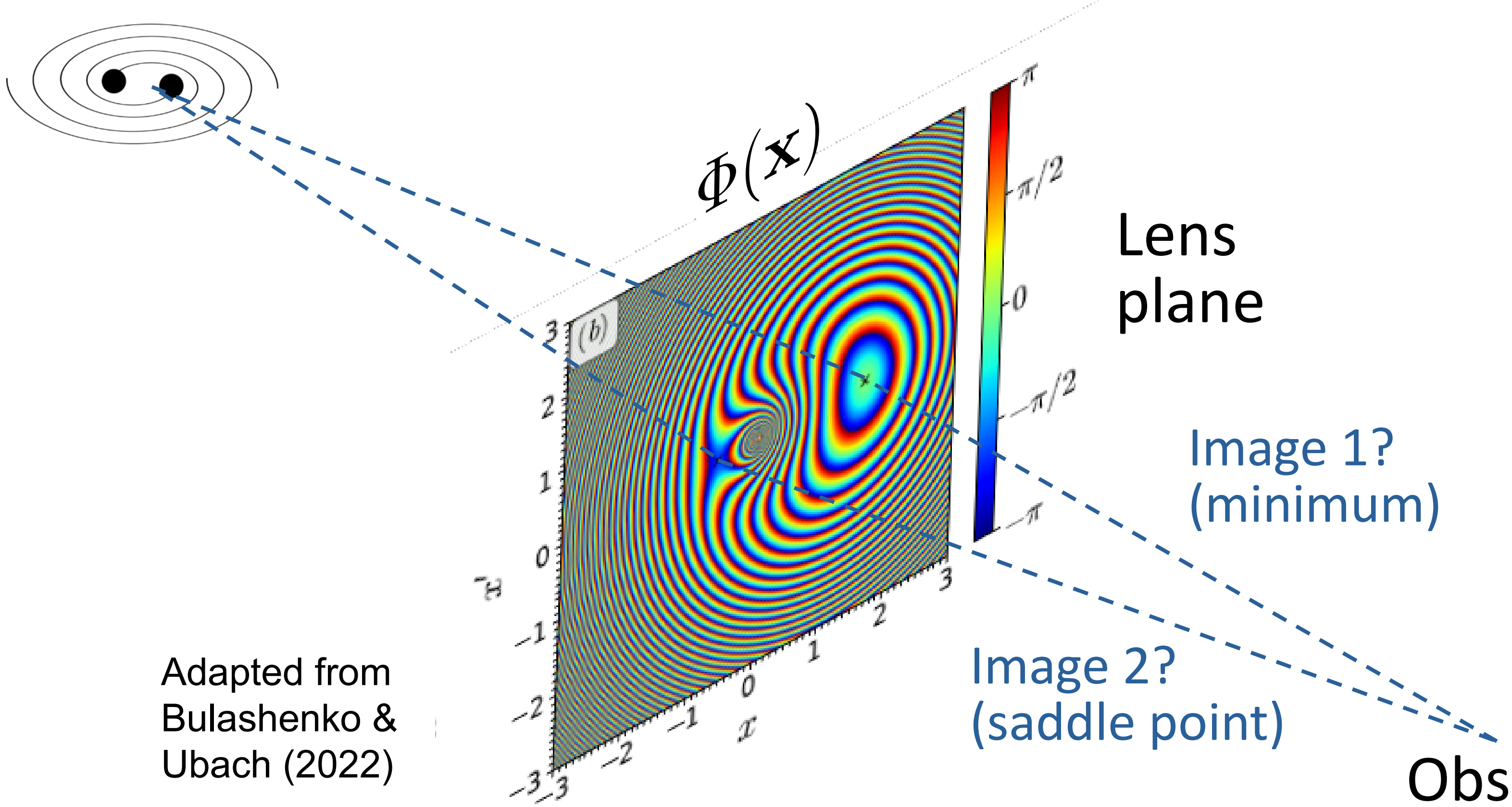
$$(\nu \gg 1)$$

$$y = 1$$

Geometric Optics approximation
(Stationary Phase Approximation of the Fresnel-Kirchhoff integral)

Point mass lens model

1. Why do wave effects appear?



Source

Lens plane

Image 1?
(minimum)

Image 2?
(saddle point)

Observer

Adapted from
Bulashenko &
Ubach (2022)

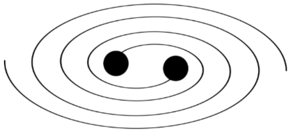
$$\nu = 4$$

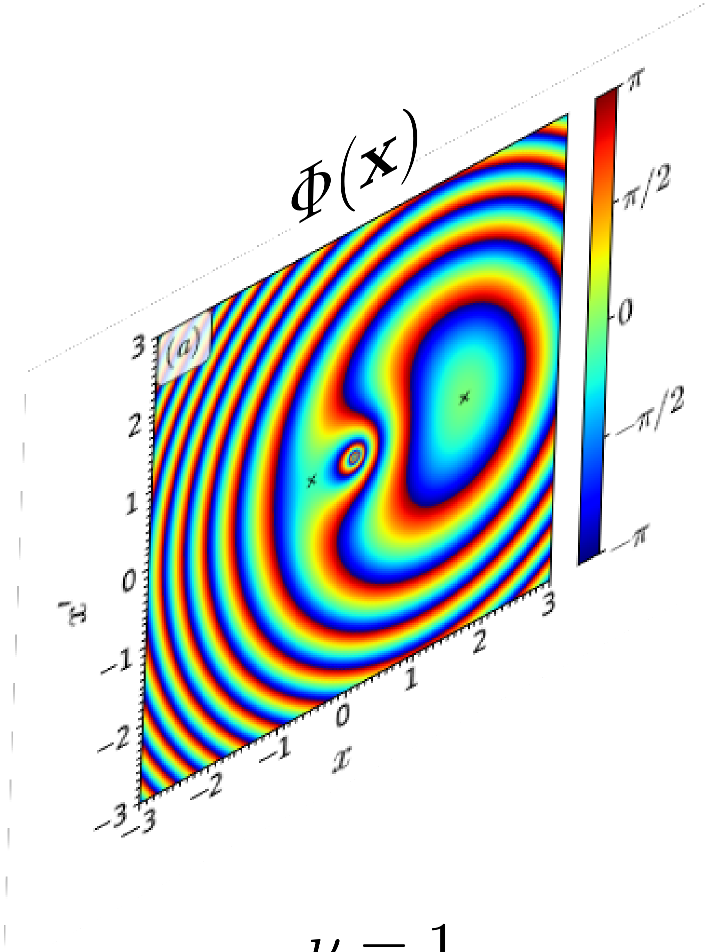
$$y = 1$$

Point mass lens model

1. Why do wave effects appear?

Source





Lens plane

Adapted from Bulashenko & Ubach (2022)

Observer

$$\nu = 1$$

$$(\nu \sim 1)$$

$$y = 1$$

Point mass lens model

1. Why do wave effects appear?

Transmission factor: $F = \dfrac{\widetilde{\varphi}(P)}{\widetilde{\varphi}_0(P)}$ $\quad \widetilde{\varphi}(P) = -\mathrm{i}\nu \dfrac{A}{d_S} e^{\mathrm{i}k(r_0+s_0)} \iint e^{2\pi\mathrm{i}\,\nu\,\tau(\mathbf{x},\mathbf{y})}\, d^2\mathbf{x}$

$$F = -\mathrm{i}\,\nu \iint e^{2\pi\mathrm{i}\,\nu\,\tau(\mathbf{x},\mathbf{y})}\, d^2\mathbf{x} \qquad \tau(\mathbf{x},\mathbf{y}) = \frac{1}{2}(\mathbf{x}-\mathbf{y})^2 - \psi(\mathbf{x}) + \psi_0$$

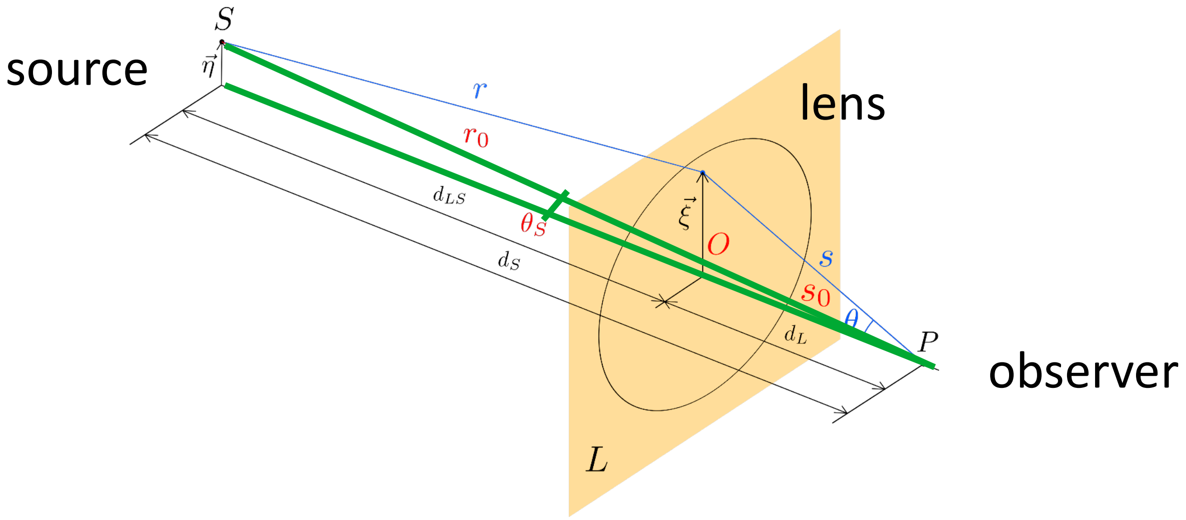
analytical solution
to Fresnel-Kirchhoff integral
(W. Gordon, 1928)

$$\psi(\mathbf{x}) = \ln|\mathbf{x}|$$

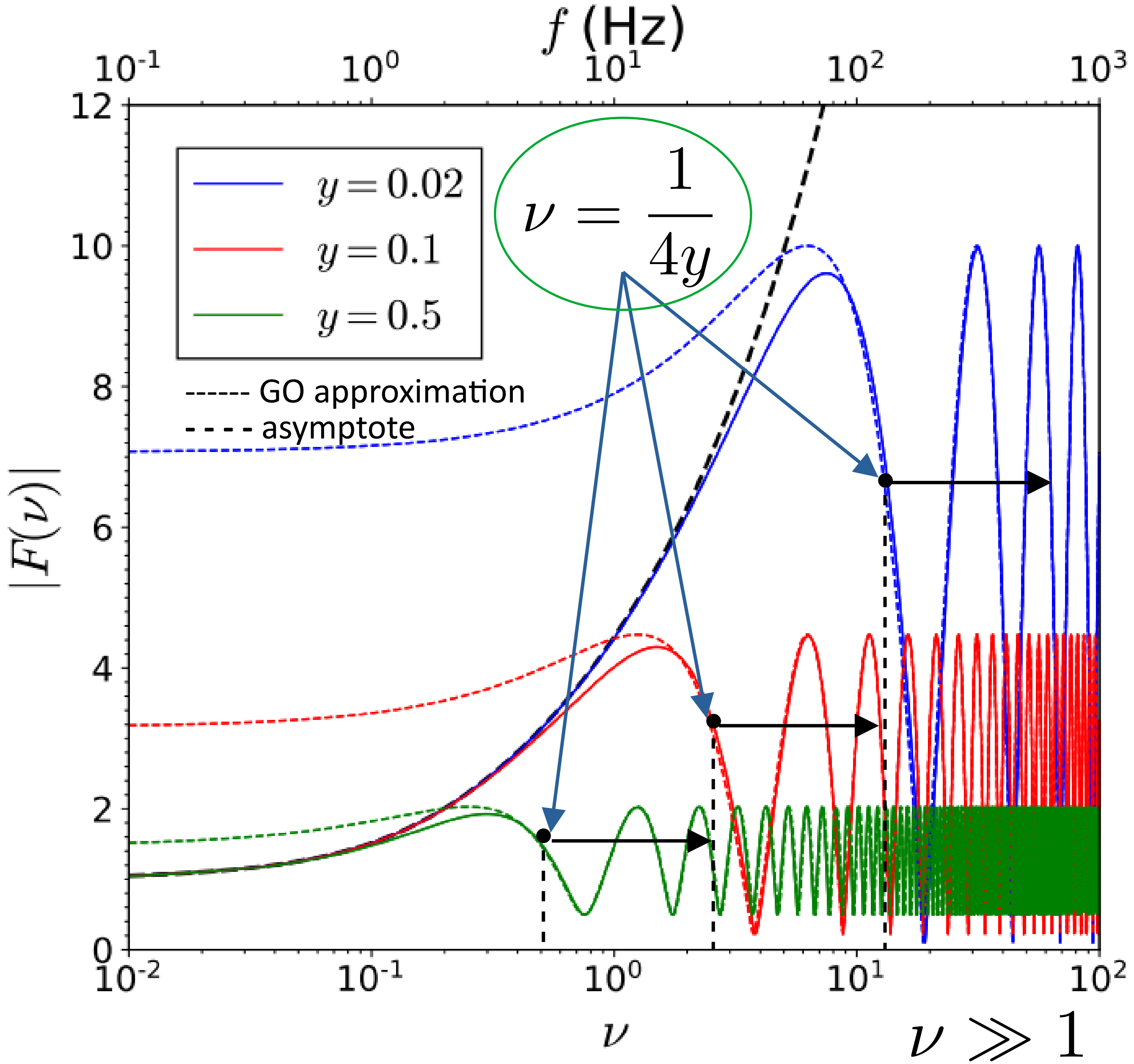
$$F(\nu, y) = e^{\frac{1}{2}\pi^2\nu} e^{\mathrm{i}\pi\nu\ln(\pi\nu)}\,\Gamma(1-\mathrm{i}\pi\nu)\,{}_1F_1(\mathrm{i}\pi\nu;\,1;\,\mathrm{i}\pi\nu y^2)$$

Point mass lens model

$$\nu = \frac{2R_\mathrm{S}}{\lambda} \qquad y = \frac{\theta_\mathrm{S}}{\theta_\mathrm{E}}$$



2. When do wave effects appear?



$\nu \sim 1$

$\nu \gg 1$
$\lambda \ll R_\mathrm{S}$

Takahashi and Nakamura (2003)
Bulashenko and Ubach (2022)

GO approximation valid:

$\cancel{\nu \gg 1}$ **?**

$$\nu \gtrsim \frac{1}{4y}$$

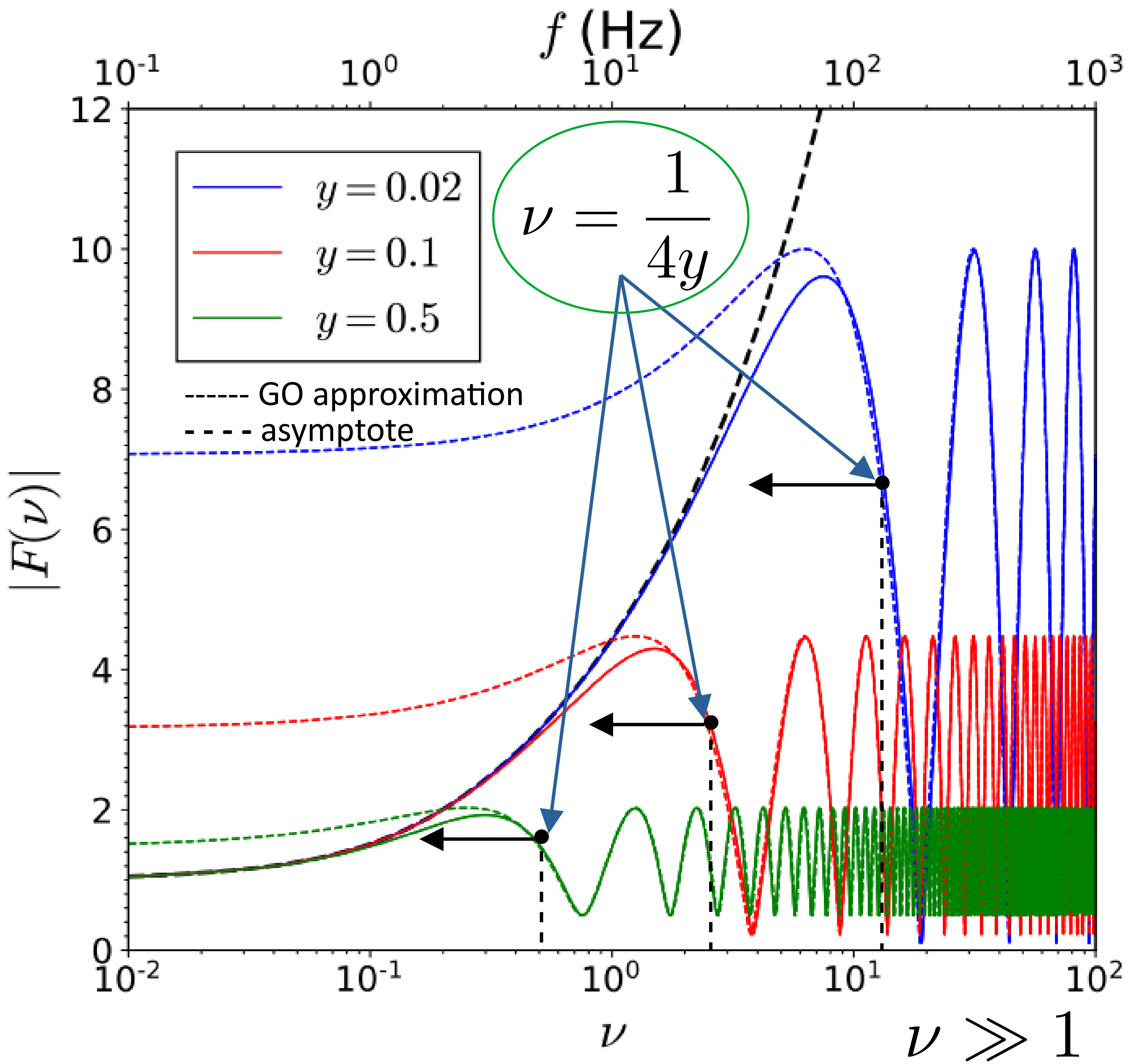
$$\Downarrow$$

$$\cancel{\lambda \ll R_\mathrm{S}}$$

$$\lambda \lesssim 8R_\mathrm{S}y$$

$$1/f_\mathrm{GW} \lesssim \tau_{21}$$

2. When do wave effects appear?



$\nu \sim 1$

$\nu \gg 1$

$\lambda \ll R_S$

Takahashi and Nakamura (2003)
Bulashenko and Ubach (2022)

Diffraction valid:

$$\cancel{\nu \lesssim 1}$$

$$\nu \lesssim \frac{1}{4y}$$

$$\Downarrow$$

$$\cancel{\lambda \gtrsim R_S}$$

$$\lambda \gtrsim 8 R_S y$$

$$1/f_{\rm GW} \gtrsim \tau_{21}$$

2. When do wave effects appear?

For our astrophysical case of interest:
Binary mergers emitting gravitational waves

$$\nu \lesssim \frac{1}{4y}$$

$$\nu = f\,\frac{4G\boxed{M_{\mathrm{Lens}}}}{c} \lesssim \frac{1}{4y} \longrightarrow \boxed{\frac{M_{\mathrm{Lens}}}{M_{\mathrm{Source}}} \gtrsim \frac{1}{y}}$$

$$f_{\mathrm{Ringdown}} \simeq 1.2 \times 10^{4}\ \mathrm{Hz}\left(\frac{M_{\odot}}{\boxed{M_{\mathrm{Source}}}}\right)$$

Source: compact binary merger
(Ringdown = last stage of merger)

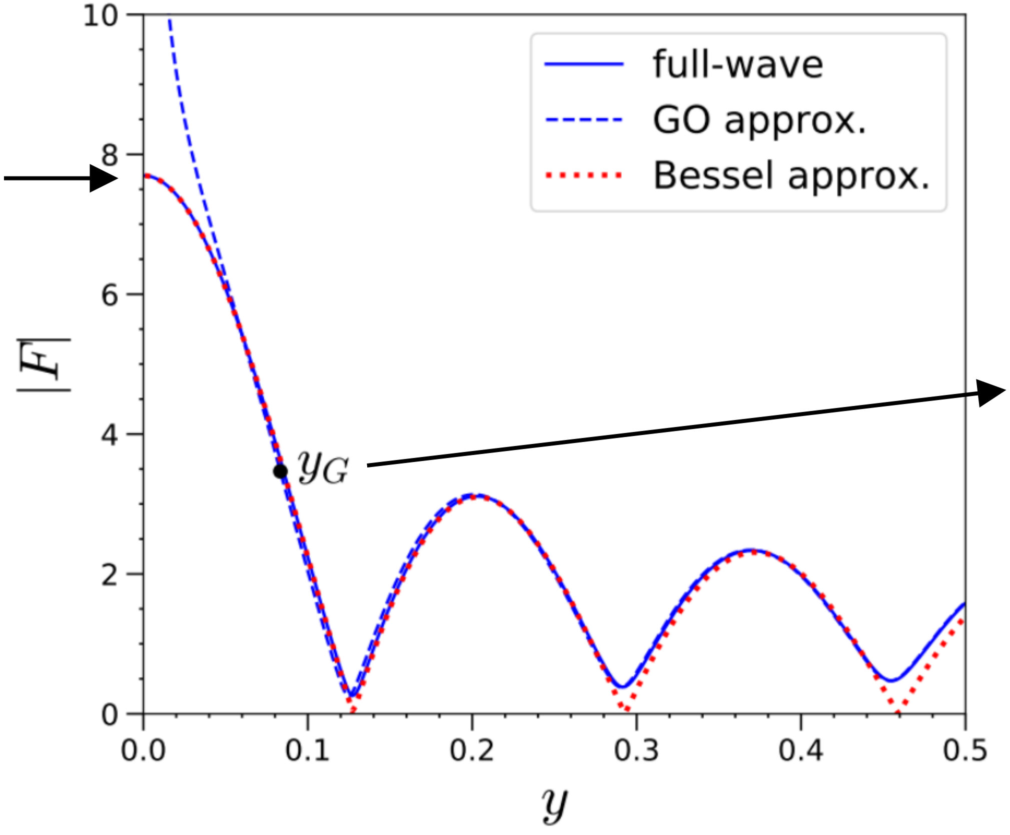
Bulashenko and Ubach (2022)

2. When do wave effects appear?

$$\boxed{\frac{M_{\rm Lens}}{M_{\rm Source}} \gtrsim \frac{1}{y}} \quad \begin{cases} y \sim 1 \longrightarrow M_{\rm Lens} \gtrsim M_{\rm Source} \\ y \ll 1 \longrightarrow M_{\rm Lens} \gg M_{\rm Source} \end{cases}$$

Close to caustics, wave effects are also important, even for large $M_{\rm Lens}$

$y \ll 1$

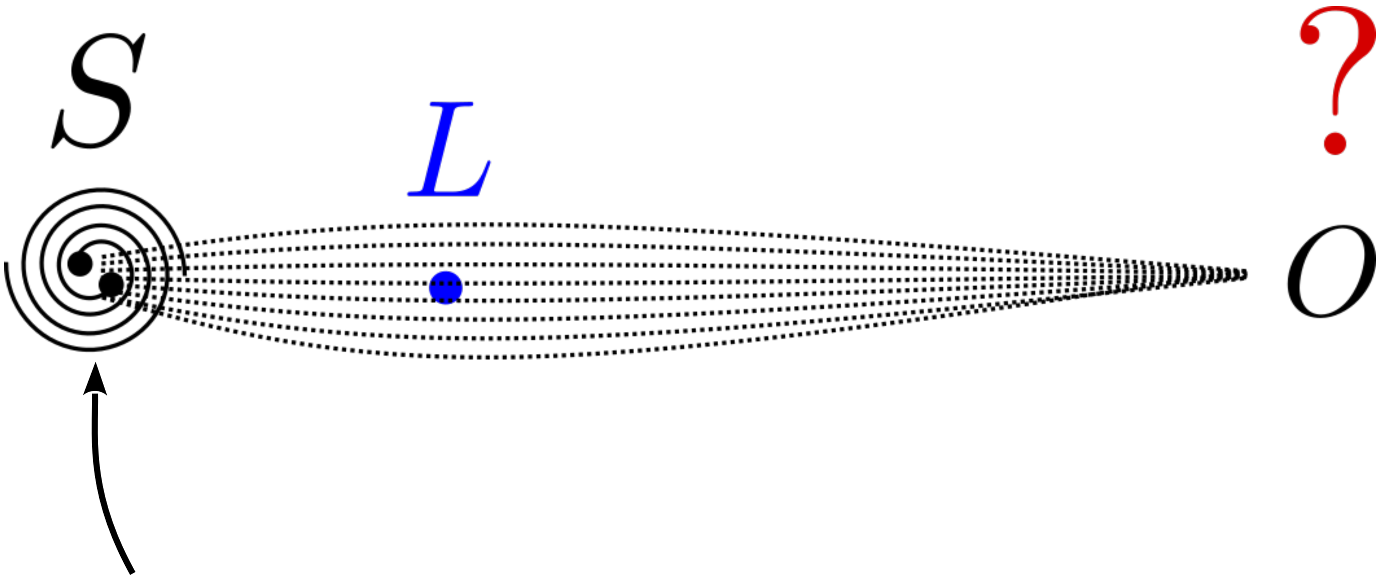


Diffraction

$$y_G \sim \frac{M_{\rm Source}}{M_{\rm Lens}}$$

Deguchi and Watson (1986)
Bulashenko and Ubach (2022)

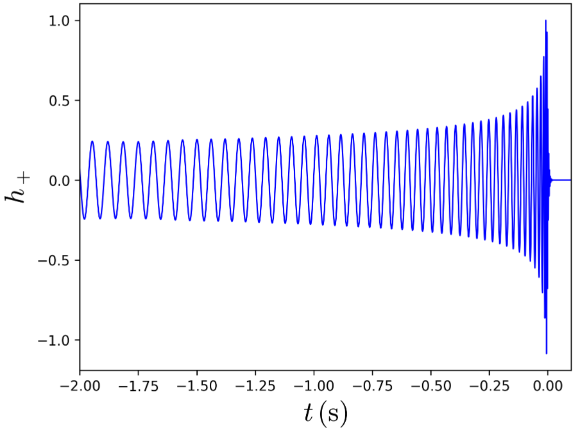
3. How do wave effects look like?



Focus on compact binary mergers
(LVK frequencies)

3. How do wave effects look like?

Compact binary merger

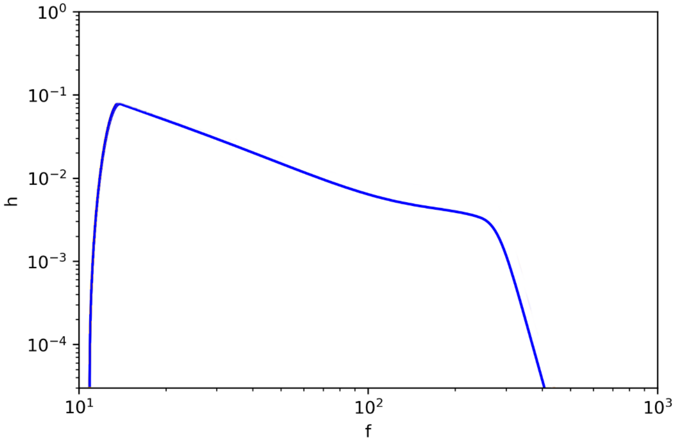


$t(\mathrm{s})$

$$h_{\text{unlensed}}(t)$$

Fourier Transform

$\longrightarrow$

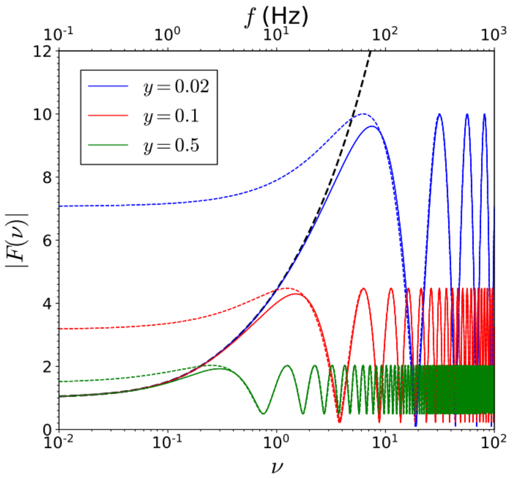


$f\,(\mathrm{Hz})$

$$\tilde{h}_{\text{unlensed}}(f)$$

$$\times$$

$$|F(f)|$$



$\longrightarrow$

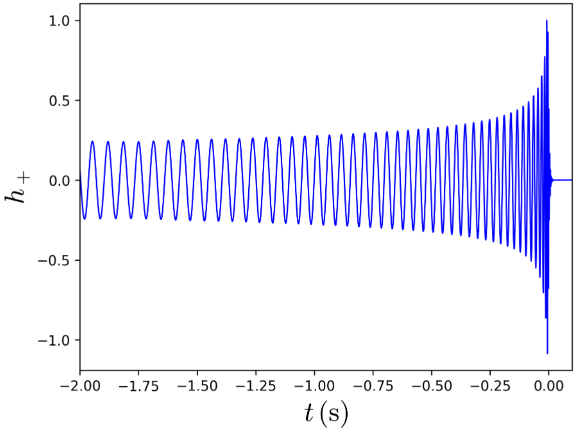
Inverse Fourier Transform

?

$$h_{\text{lensed}}(t)$$

3. How do wave effects look like?

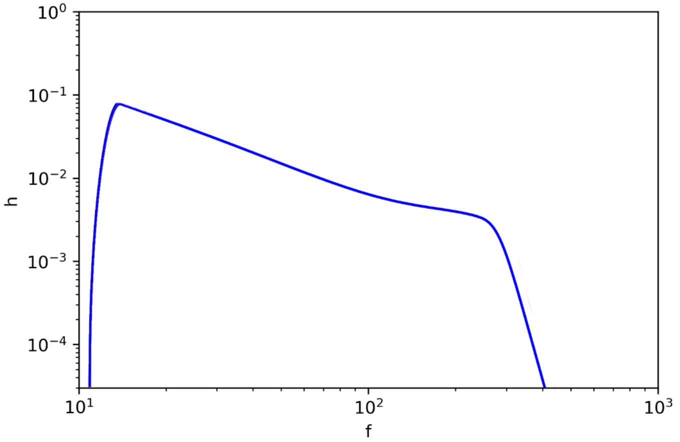
Compact binary merger



$t(\mathrm{s})$

Fourier Transform

$\longrightarrow$



$f\,(\mathrm{Hz})$

?

$$h_{\mathrm{unlensed}}(t)$$

$$\widetilde{h}_{\mathrm{unlensed}}(f)$$

$$\times$$

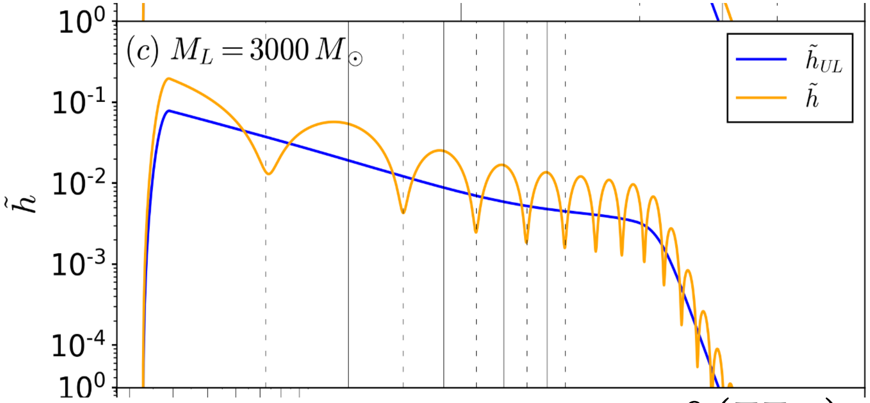
$$|F(f)|$$

$\longrightarrow$

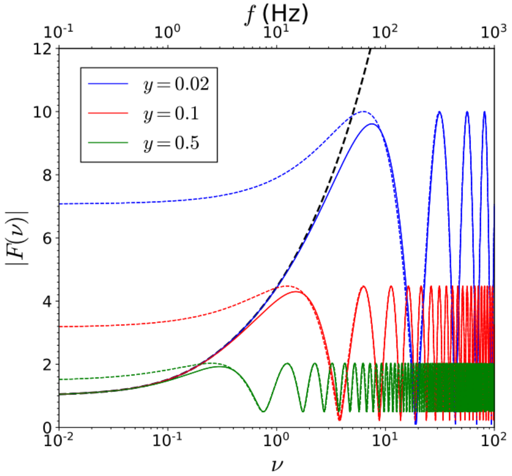
Inverse Fourier Transform

$$h_{\mathrm{lensed}}(t)$$

$$\boxed{|\widetilde{h}_{\mathrm{lensed}}(f)| = |\widetilde{h}_{\mathrm{unlensed}}(f)| \cdot |F|(f)}$$



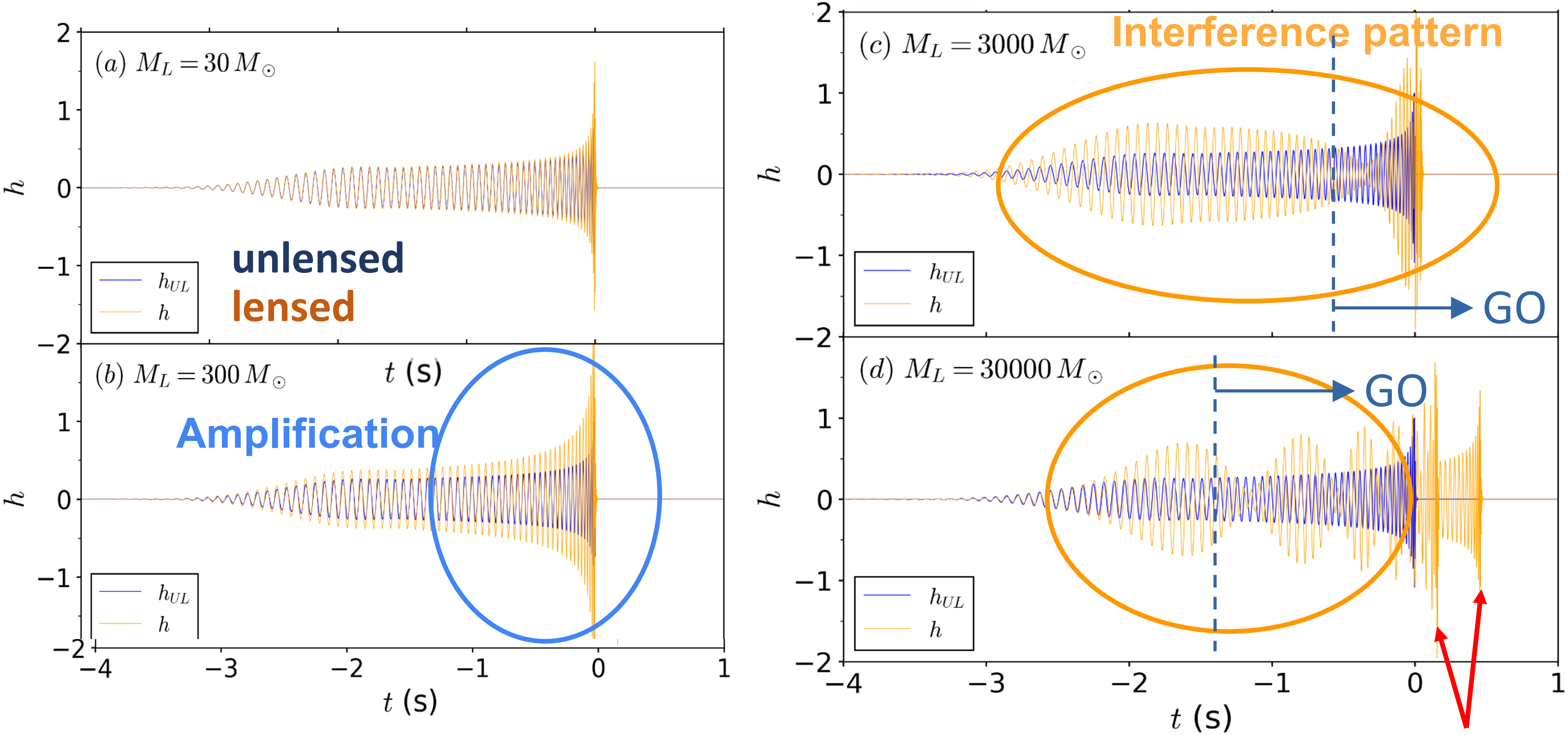
$f\,(\mathrm{Hz})$



3. How do wave effects look like?

Point mass lens model, $y = 0.25$

$\nu \downarrow$



unlensed

lensed

Amplification

Interference pattern

GO

GO

Two images

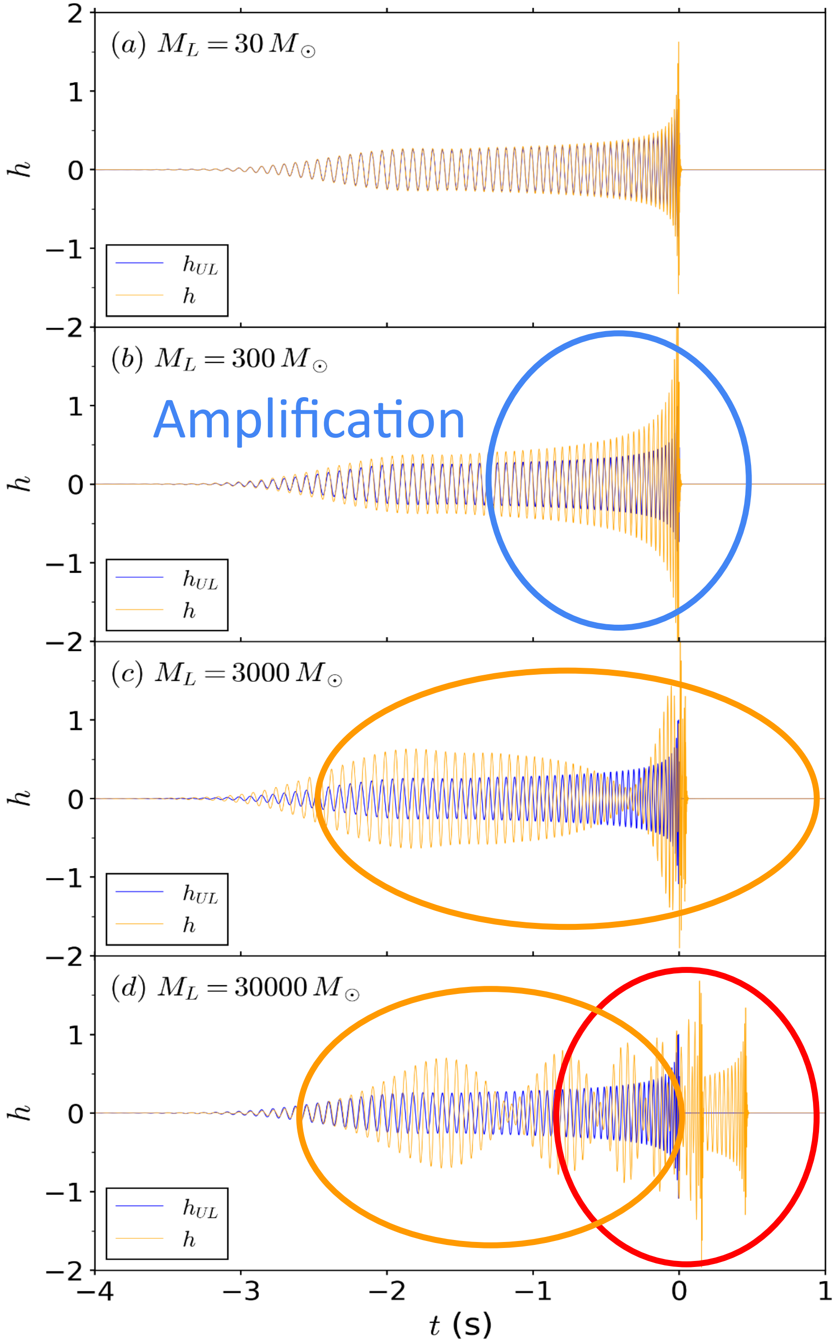
$\nu \uparrow$

Bondarescu, Ubach, Bulashenko, Lundgren (2023)

3. How do wave effects look like?

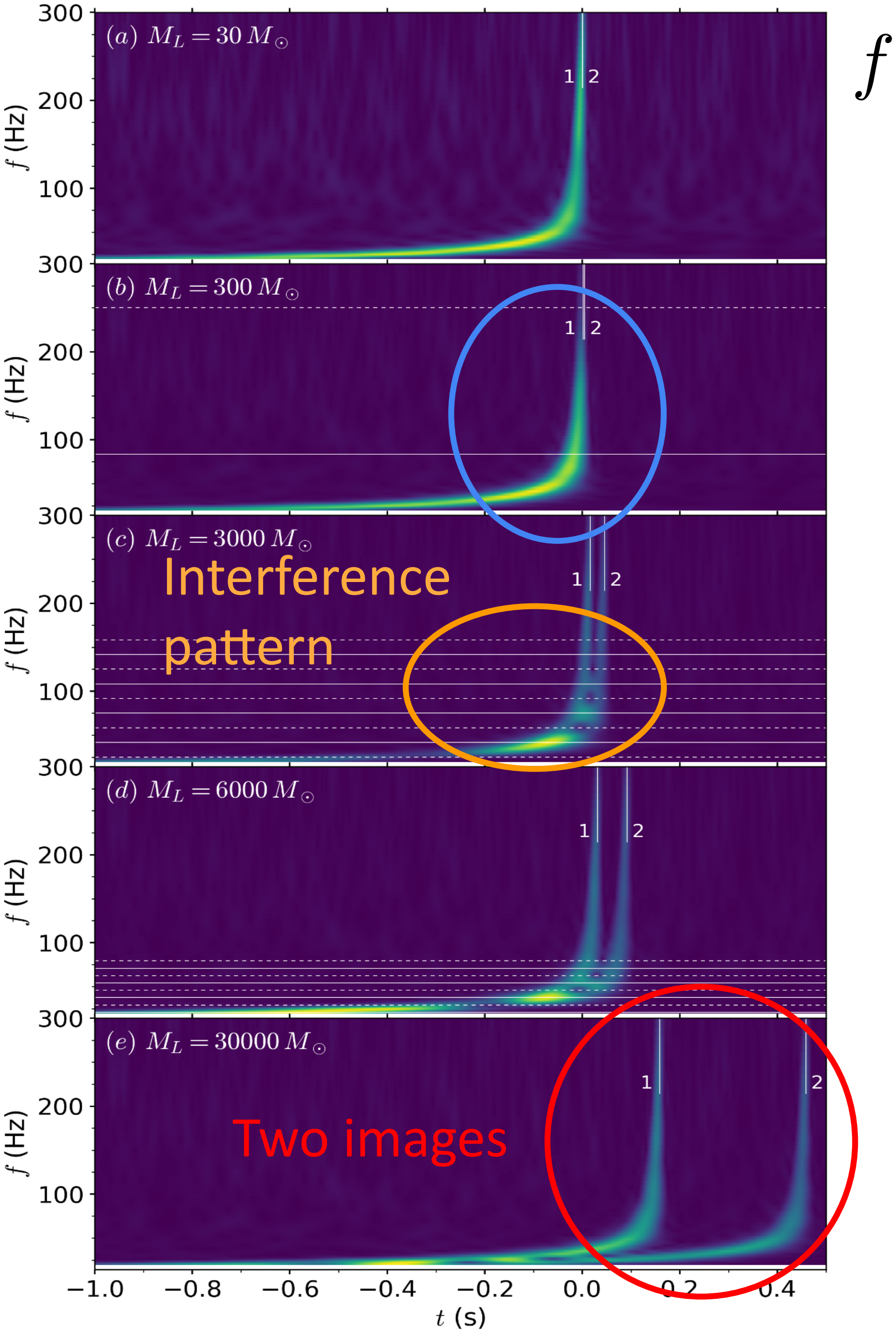
$y = 0.25$

$h(t)$



Amplification

$f(t)$



Interference pattern

Two images

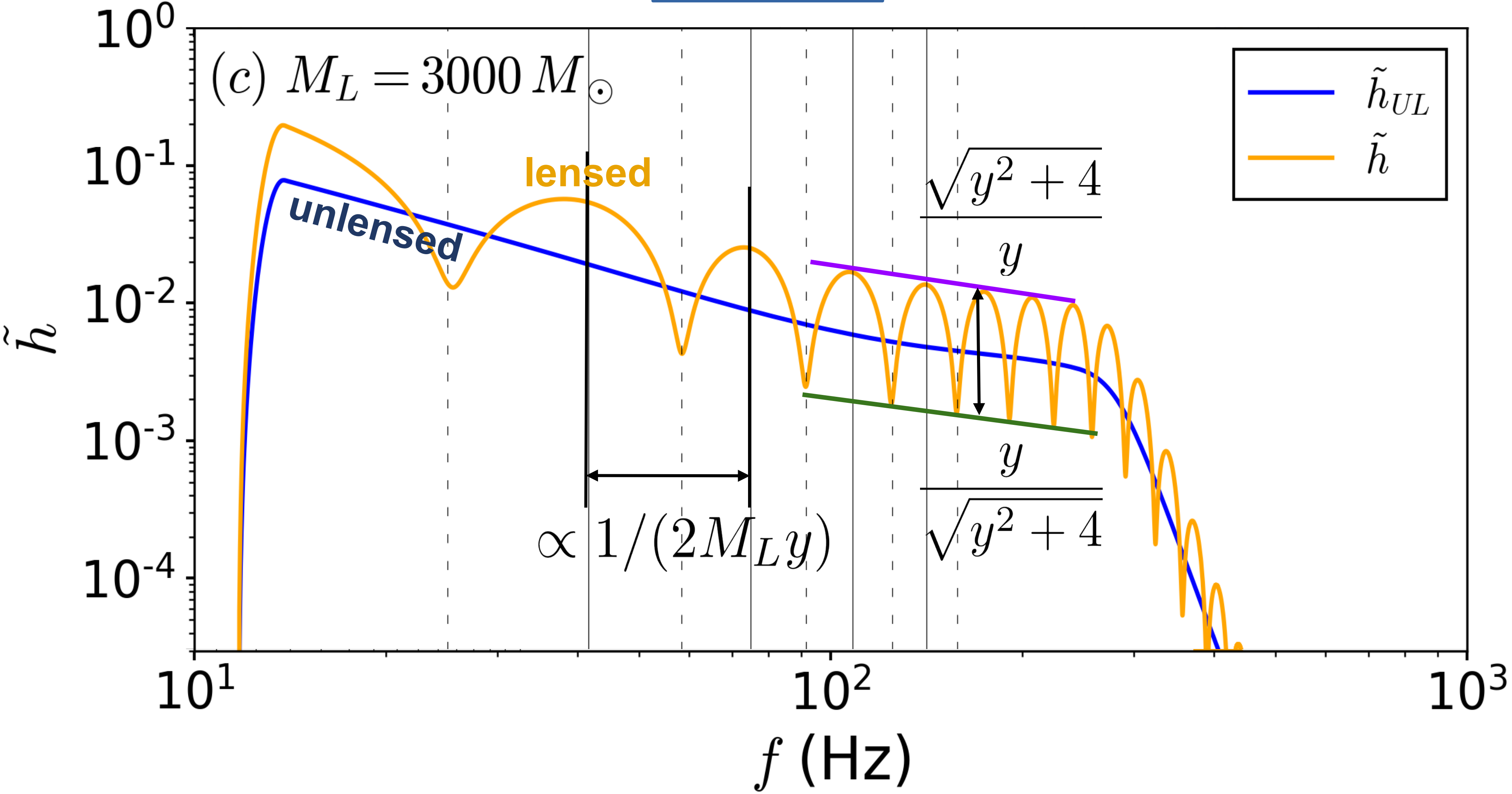
Bondarescu, Ubach, Bulashenko, Lundgren (2023)

3. How do wave effects look like?

Evenly spaced interference pattern

$y = 0.25$

$\boxed{y \lesssim 0.5}$

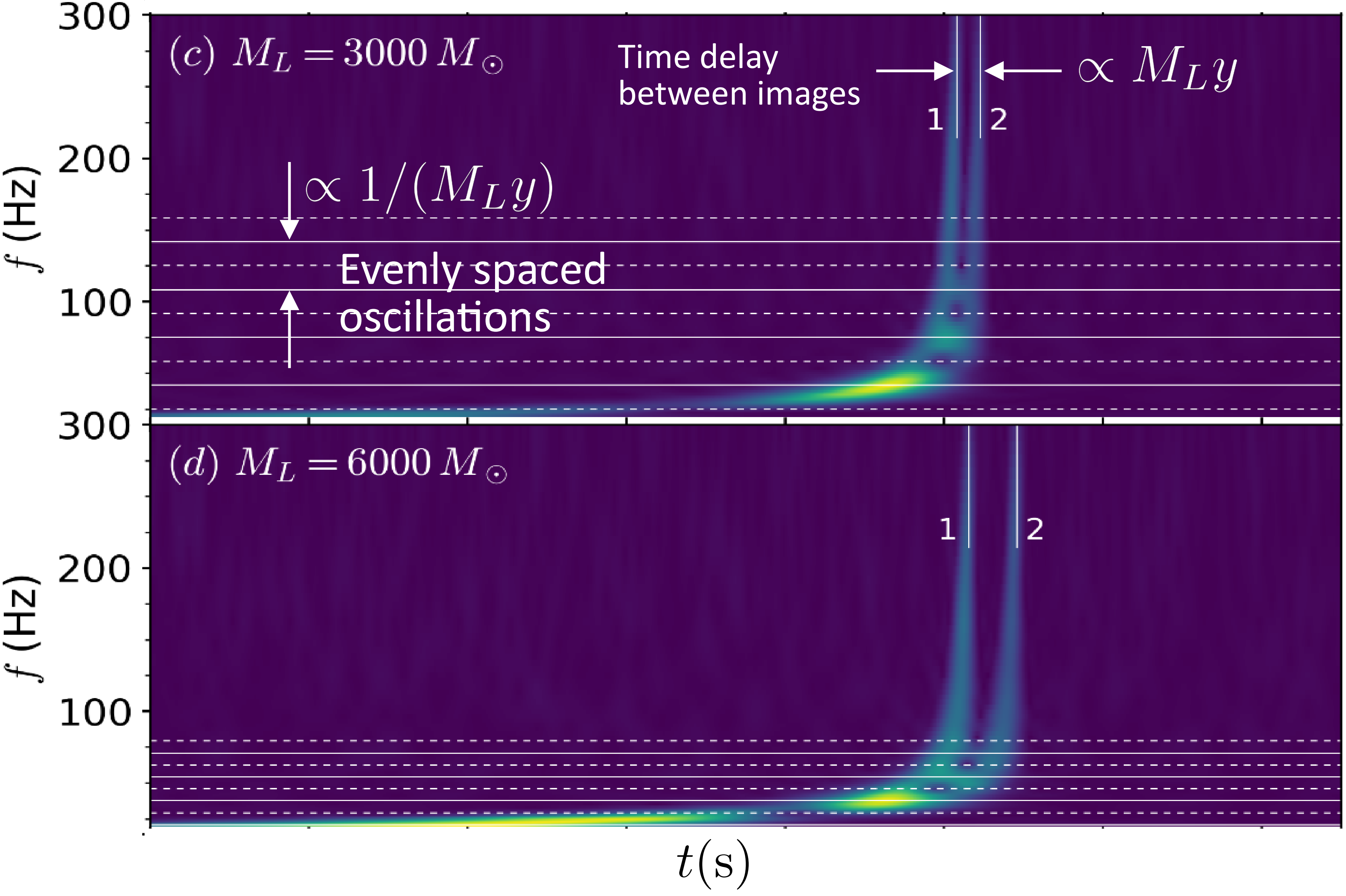


Bulashenko&
Ubach (2022)

Bondarescu, Ubach,
Bulashenko, Lundgren (2023)

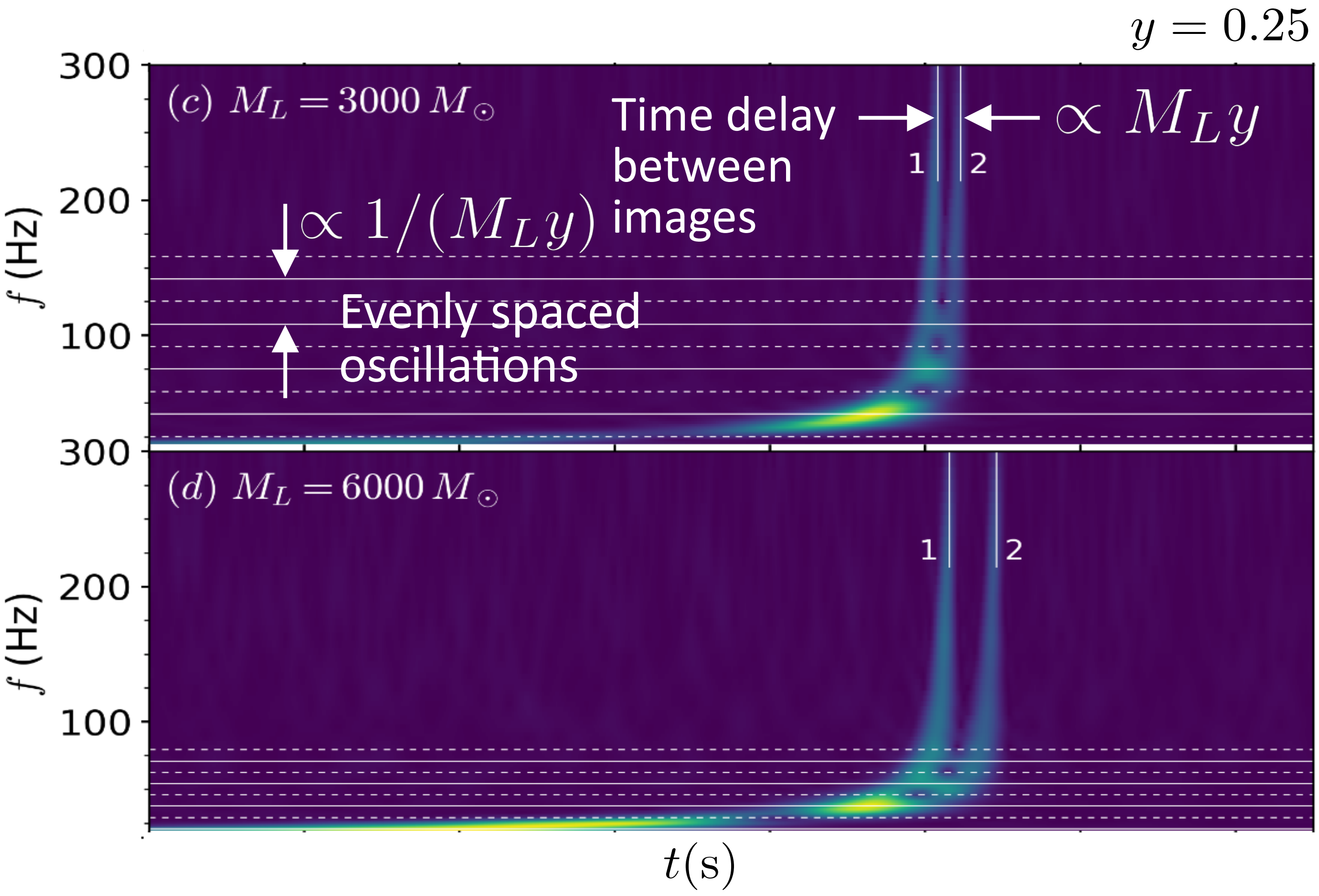
3. How do wave effects look like?

$y = 0.25$



Adapted from Bondarescu, Ubach, Bulashenko, Lundgren (2023)

3. How do wave effects look like?



Time delay between images Δt

Spacing between oscillations (consecutive maxima/minima) Δf

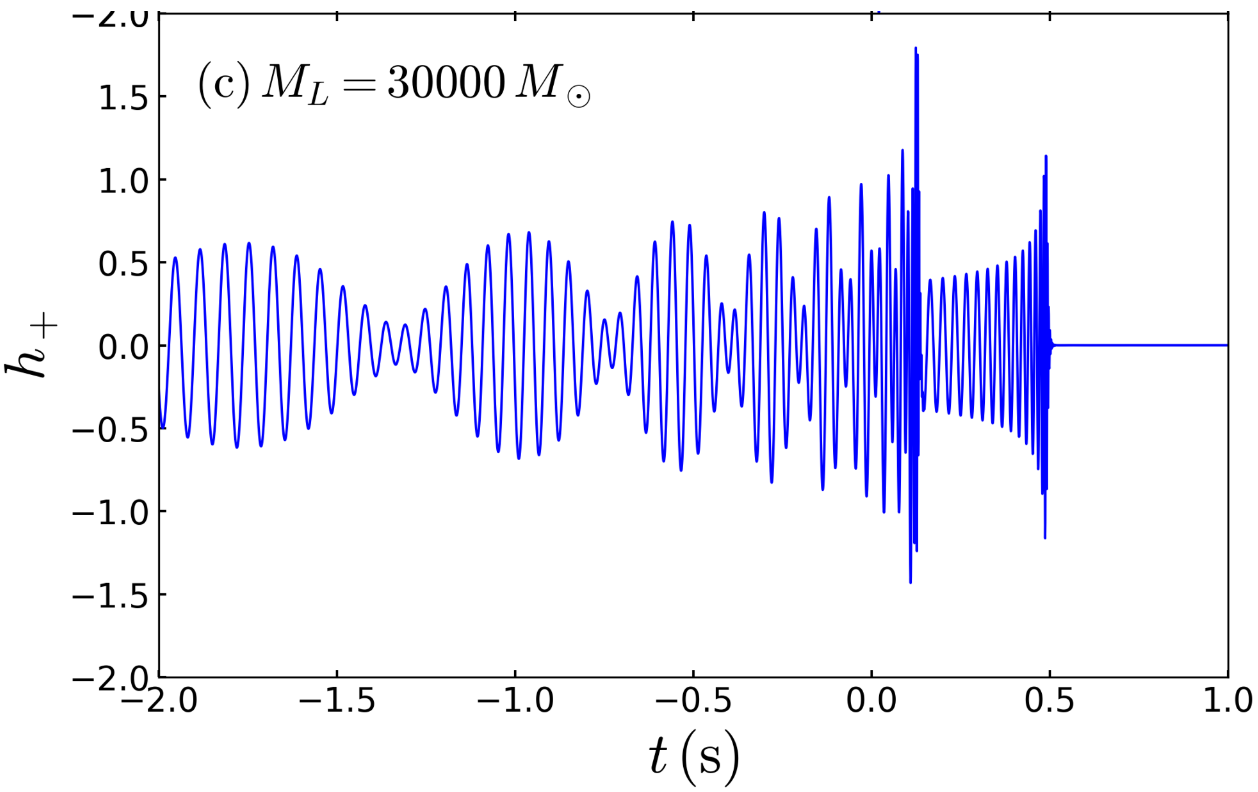
$$\boxed{\Delta t \sim \frac{1}{\Delta f}}$$

Signatures of microlensing:
Evenly spaced oscillations + Time-delayed images

→ To distinguish from eccentric, precessing and other frequency-modulated signals

Adapted from Bondarescu, Ubach, Bulashenko, Lundgren (2023)

Artistic sonification



Ubach & Espuny, arXiv:2407.09588 (2024)

https://zoom3.net/sonificacions/

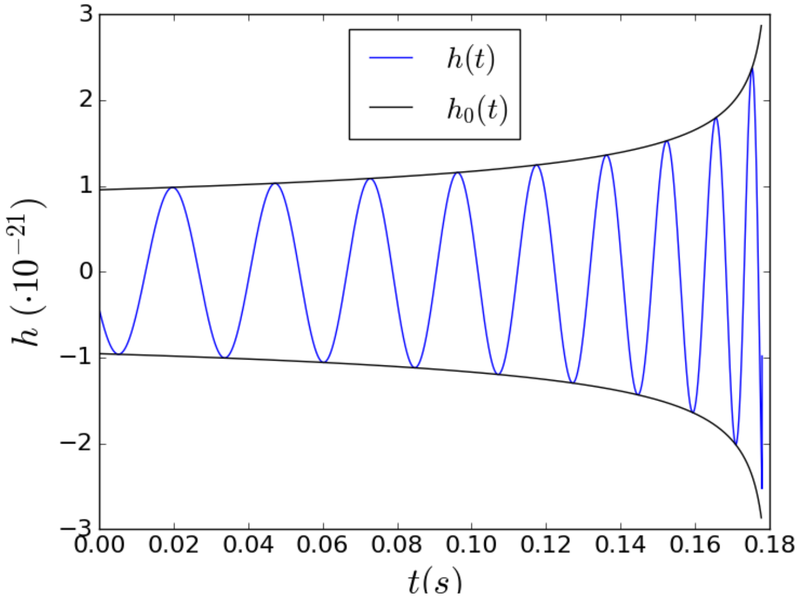
Some examples:
https://zoom3.net/sonificacions/ ⟶
ona-gravitacional-lent-exemples.html

***Geometric Optics**
***Point mass lens model**

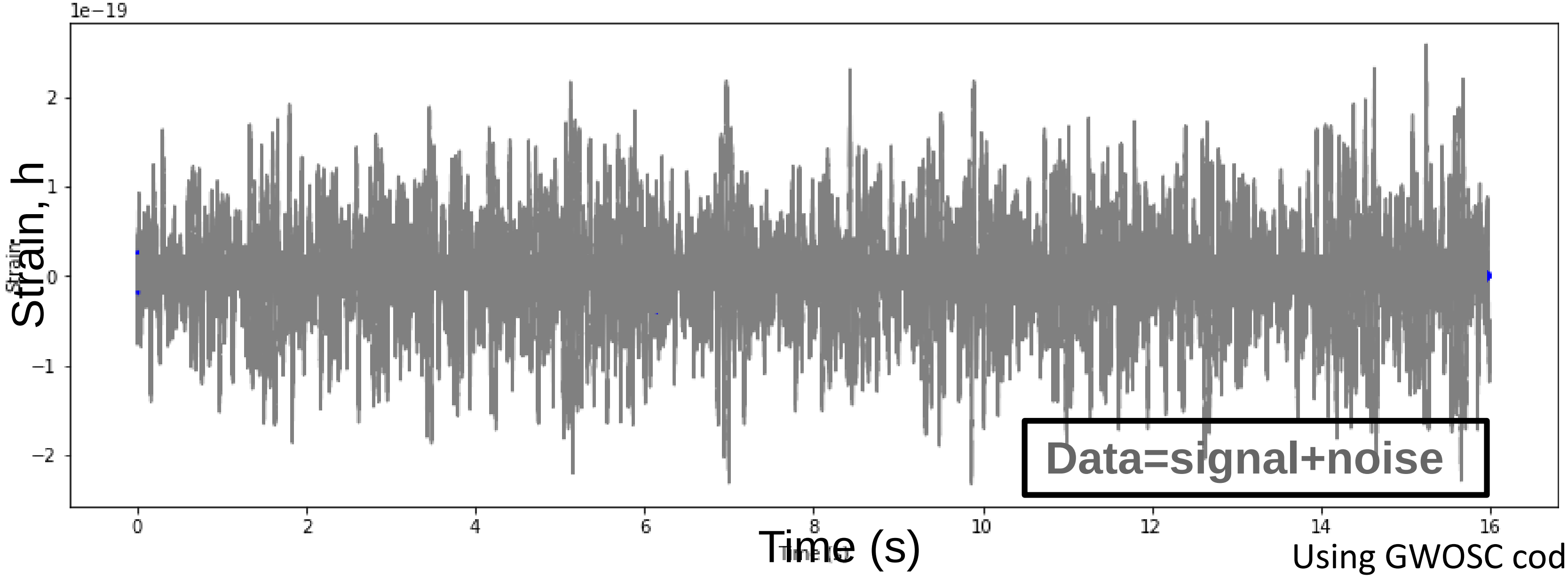
4. Are they detectable?

Detection technique: matched filtering

Template:



Recovery of an injected signal
(mock example)

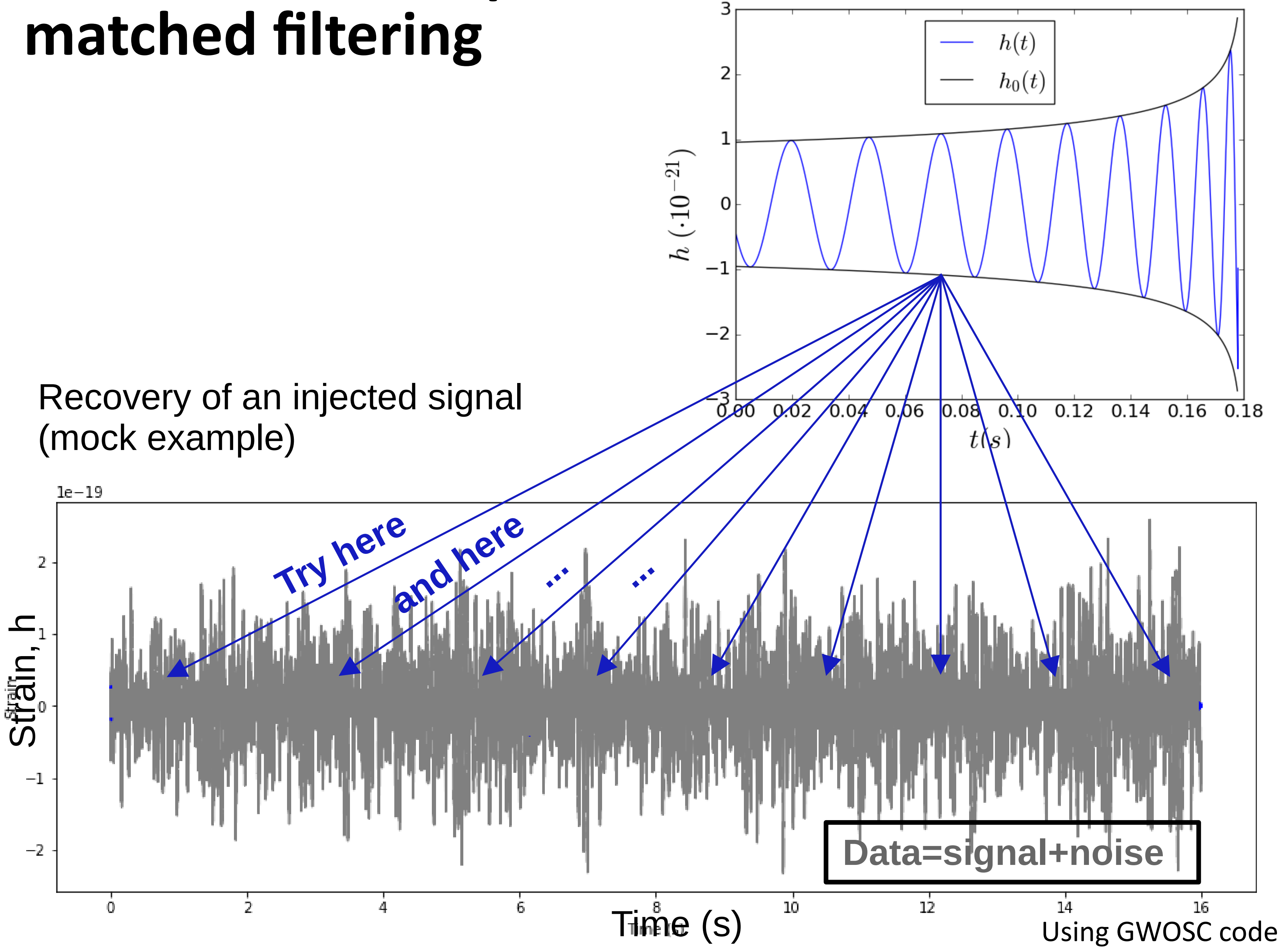


Using GWOSC code

4. Are they detectable?

Detection technique: matched filtering

Template:



Recovery of an injected signal (mock example)

Using GWOSC code

4. Are they detectable?

Detection technique: matched filtering

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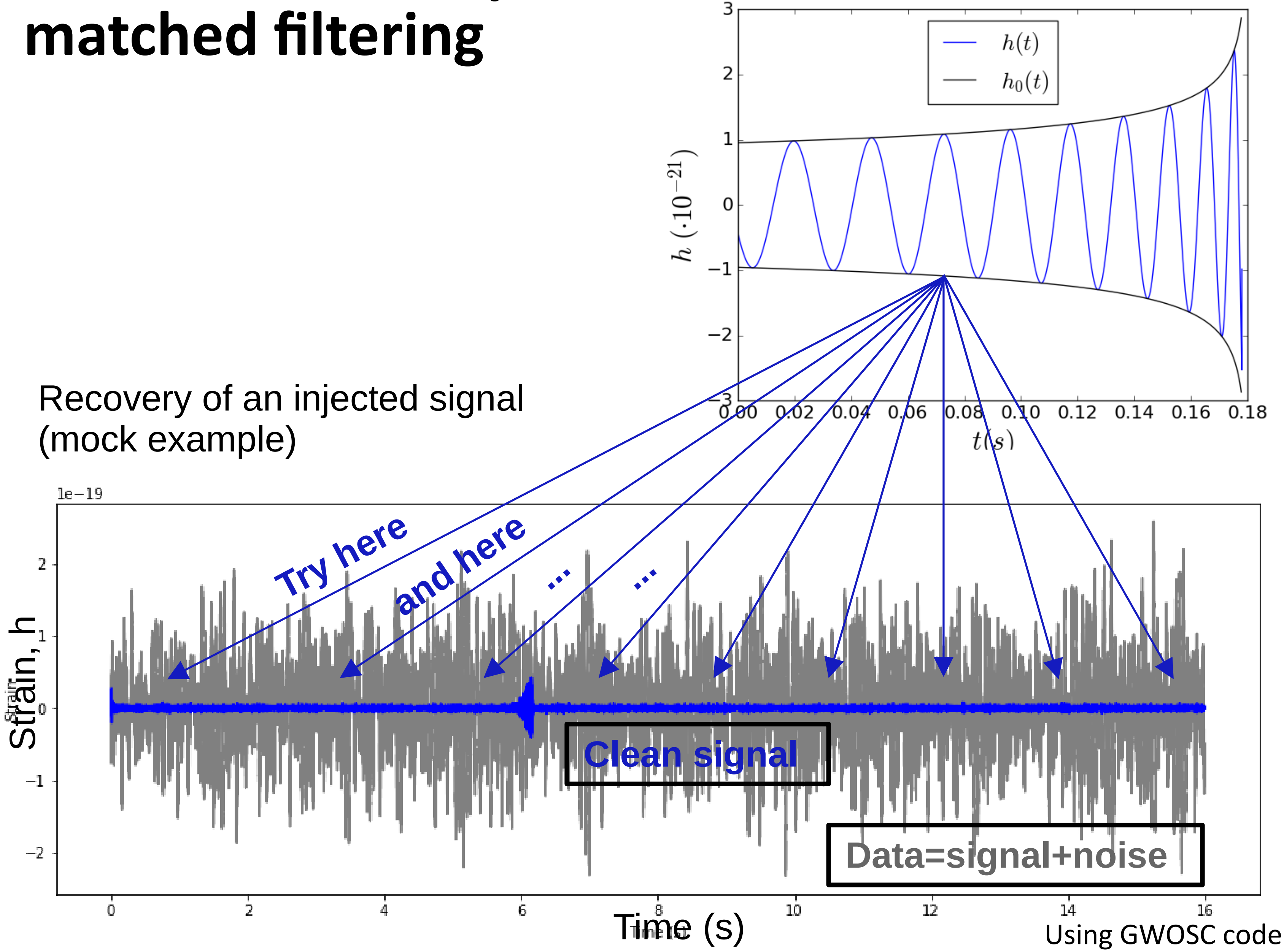


Recovery of an injected signal
(mock example)

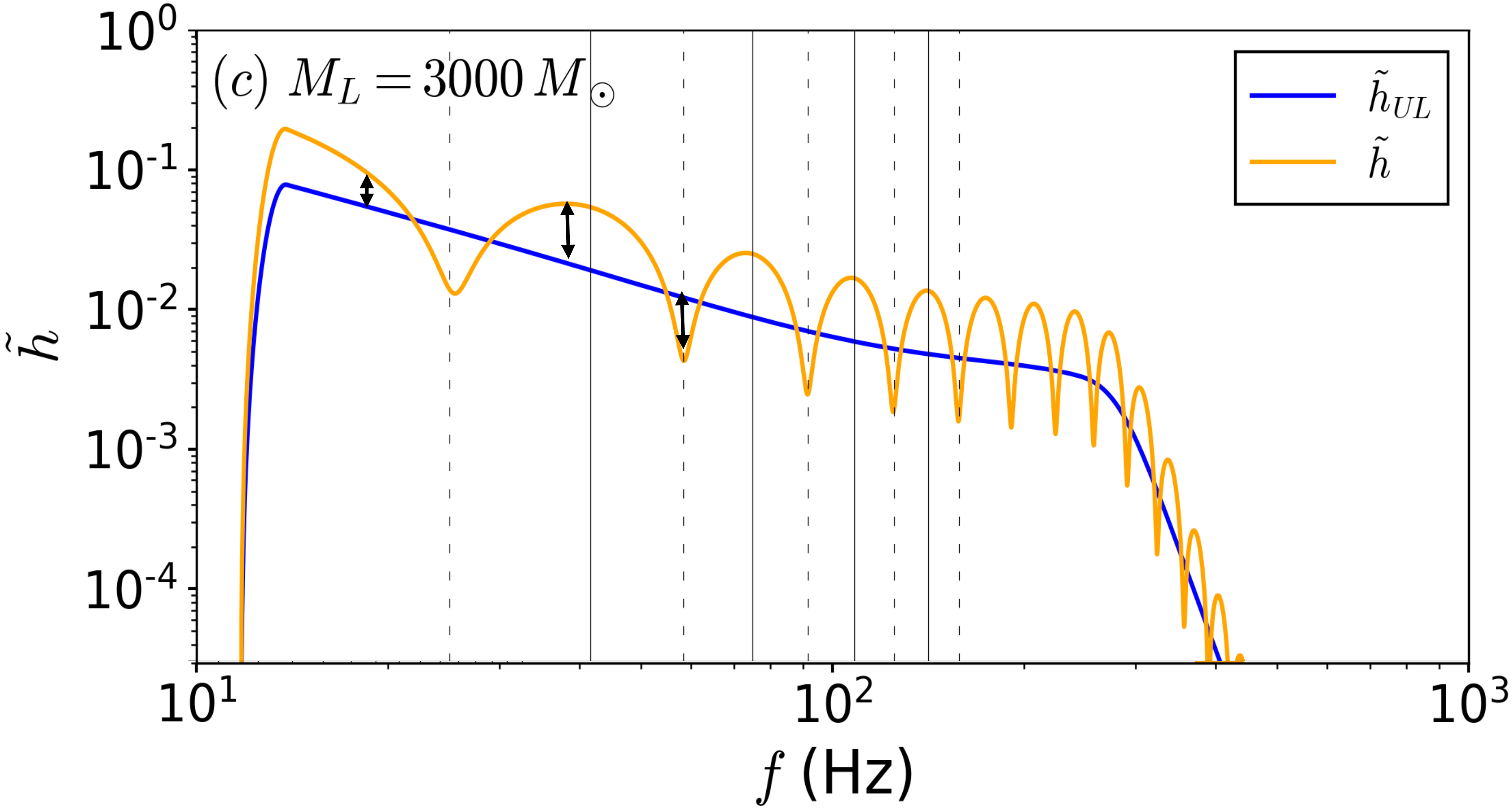
Using GWOSC code

4. Are they detectable?

Match between templates, $\boxed{\mathcal{M}}$

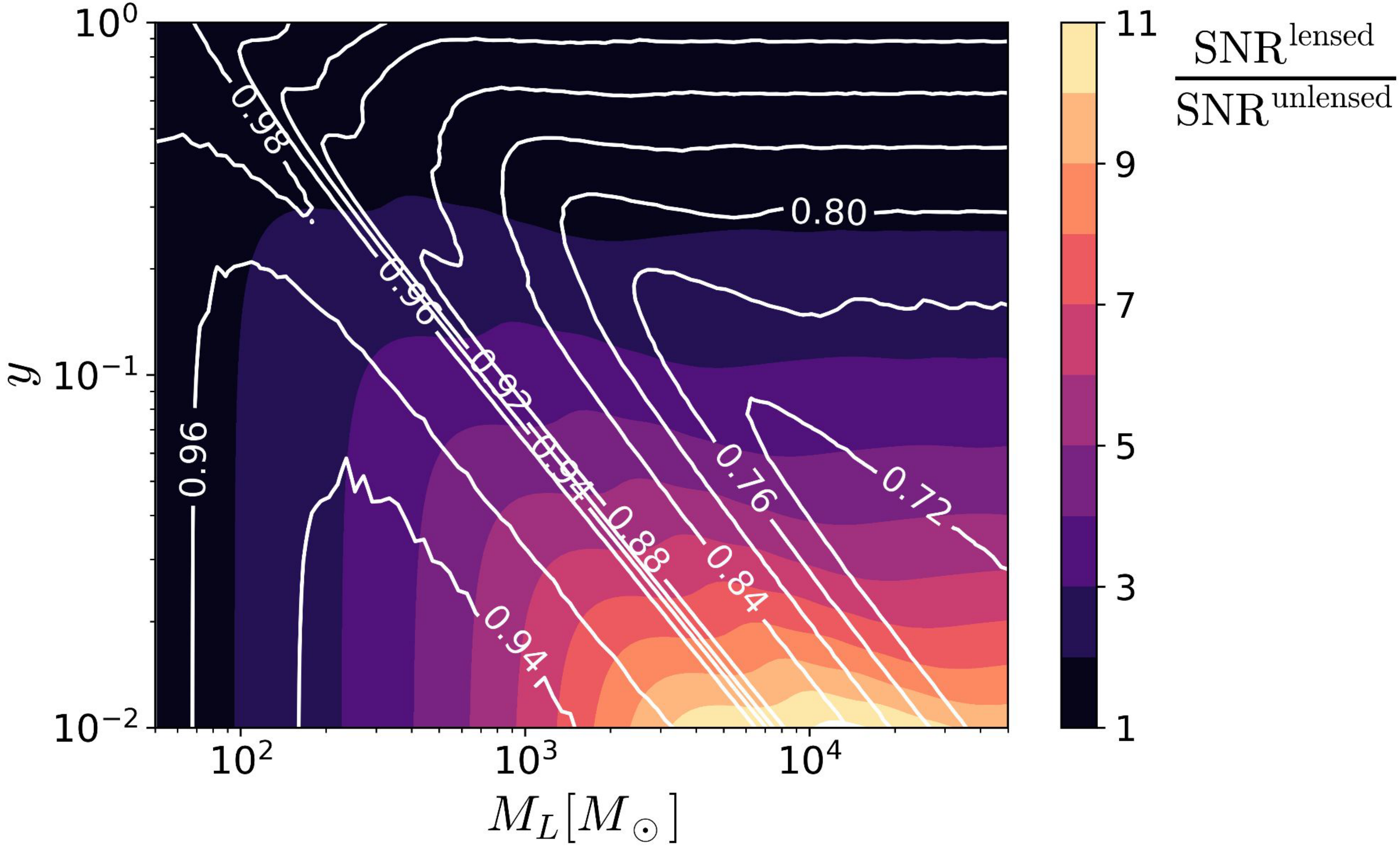
Quantifies the non-deviation on of **one template** from **another template**, integrated over frequency

$$\mathcal{M}(h, h_{UL}) = \frac{\langle f^{1/2} h_{UL}, h_{UL}\rangle}{\sqrt{\langle h_{UL}, h_{UL}\rangle \langle \sqrt{f} h_{UL}, \sqrt{f} h_{UL}\rangle}}, \langle a, b\rangle = 2\int_{f_{\min}}^{f_{\max}} \frac{\tilde{a}^*(f)\tilde{b}(f) + \tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} df$$



4. Are they detectable?

Match determines distortion from the unlensed signal

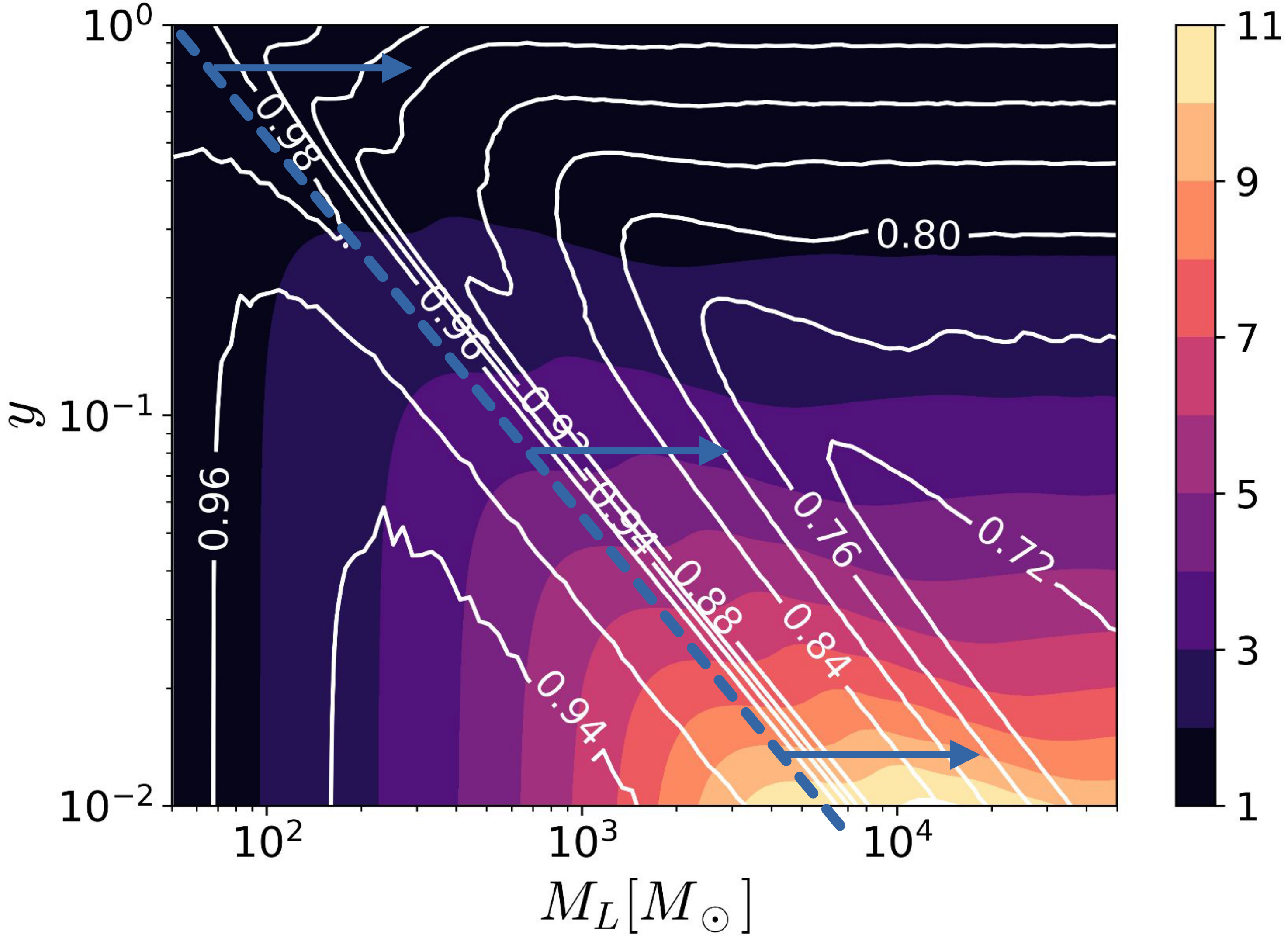


Bondarescu, Ubach, Bulashenko, Lundgren (2023)
See also Mishra+(2024)

*However, see Chan+(2024)
→ detectability might decrease in wave-optics region

4. Are they detectable?

Match determines distortion from the unlensed signal



$$\frac{\mathrm{SNR}^{\mathrm{lensed}}}{\mathrm{SNR}^{\mathrm{unlensed}}}$$

$$\boxed{\frac{M_{\mathrm{Lens}}}{M_{\mathrm{Source}}} \gtrsim \frac{1}{y}}$$

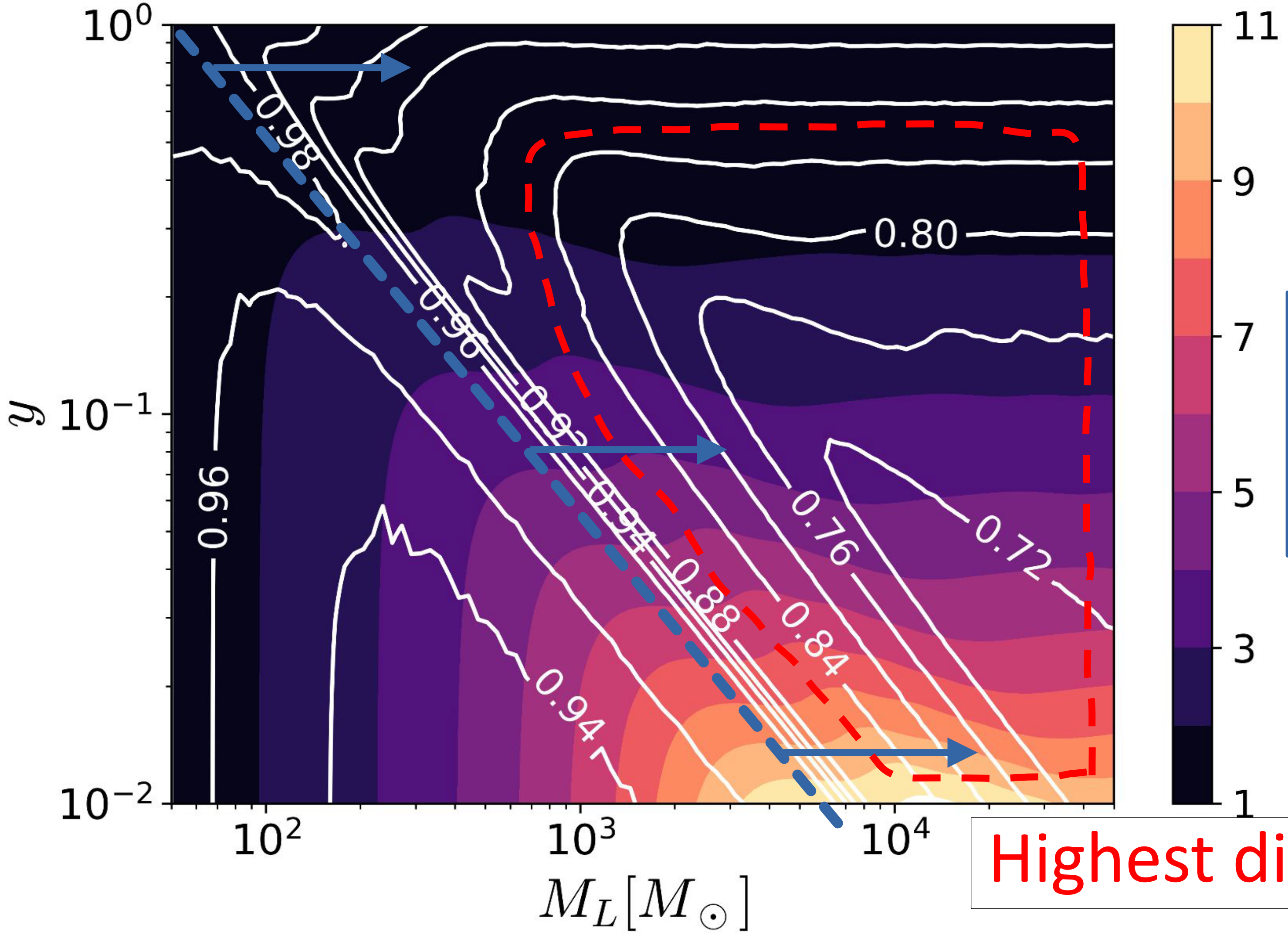
$$M_{\mathrm{Source}} \simeq 60\ M_\odot$$

Bondarescu, Ubach, Bulashenko, Lundgren (2023)
See also Mishra+(2024)

*However, see Chan+(2024)
→ detectability might decrease in wave-optics region

4. Are they detectable?

Match determines distortion from the unlensed signal



$$\frac{\mathrm{SNR}^{\mathrm{lensed}}}{\mathrm{SNR}^{\mathrm{unlensed}}}$$

$$\boxed{\frac{M_{\mathrm{Lens}}}{M_{\mathrm{Source}}} \gtrsim \frac{1}{y}}$$

$$M_{\mathrm{Source}} \simeq 60\, M_\odot$$

Highest distortion

Bondarescu, Ubach, Bulashenko, Lundgren (2023)
See also Mishra+(2024)

*However, see Chan+(2024)
→ detectability might decrease in wave-optics region

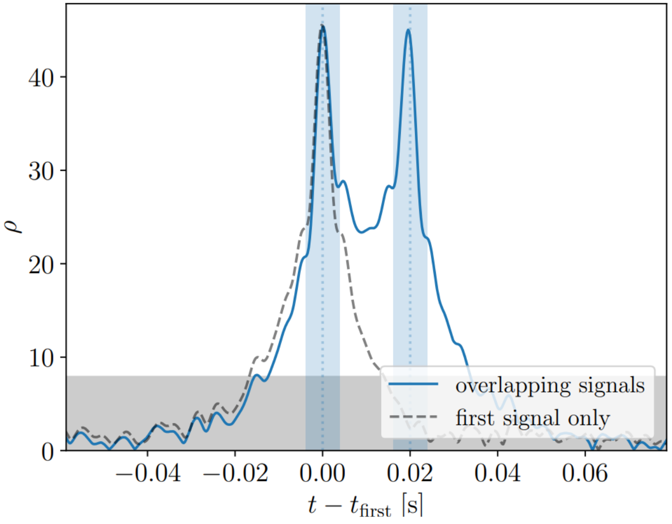
4. Are they detectable?

Alternatives or complementary tests to match filtering?

Pattern recognition,
Machine Learning...

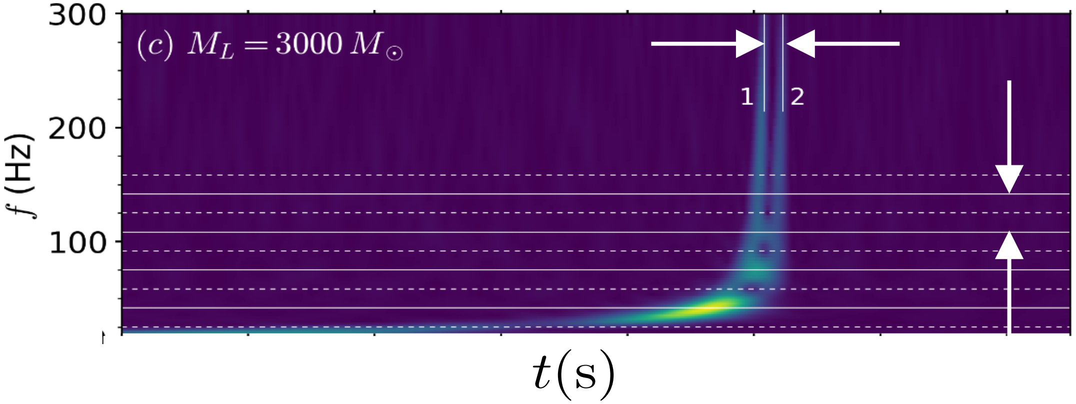
Kim+ (2021),
Bada Nerin+ (2024)

Time domain tests
(SNR,χ^2)...



Lo+ (2024)

Feature extraction?



Time delay

Evenly spaced
oscillations

Conclusions

Wave effects on gravitational lensing of gravitational waves

1. Why do they appear?

Coherent emission, long wavelength

2. When are they significant?

$$1/f_{\rm GW} \lesssim \tau_{21}\,,\ \lambda \lesssim 8R_{\rm S}y,\ \boxed{M_{\rm Lens}/M_{\rm Source} \gtrsim y}$$

3. How do they look like?

- Supressed by diffraction (look like unlensed)
- Amplification
- Interference pattern
- Multiple images

4. Are they detectable?

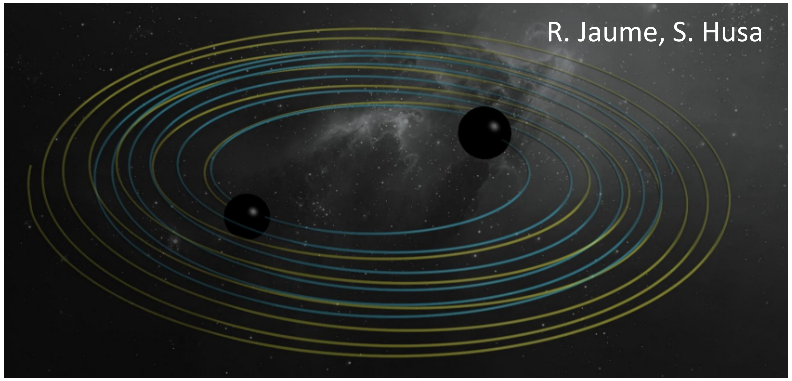
Some of them:

- Mainly for interference
- Difficult for amplification (diffraction)

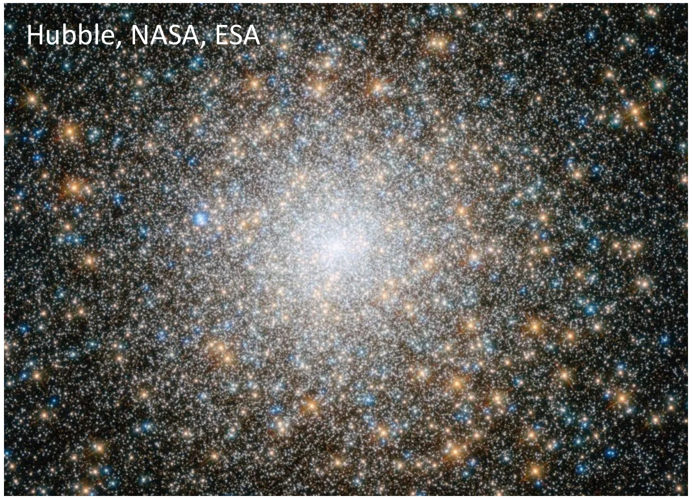
  Work in progress

Current work

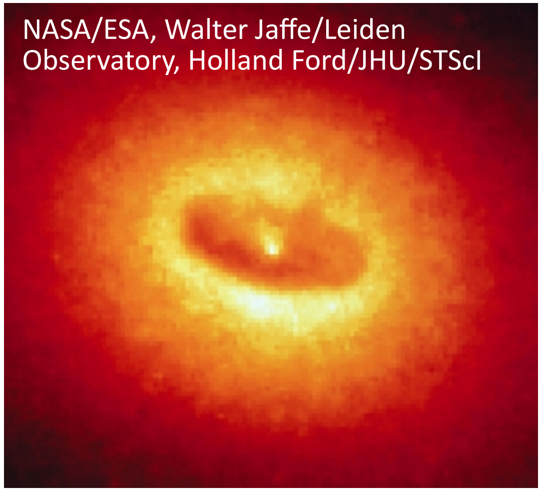
Gravitational waves can come from different environments:


R. Jaume, S. Husa

Isolated binaries, «in the field»


Hubble, NASA, ESA

Binaries in star clusters

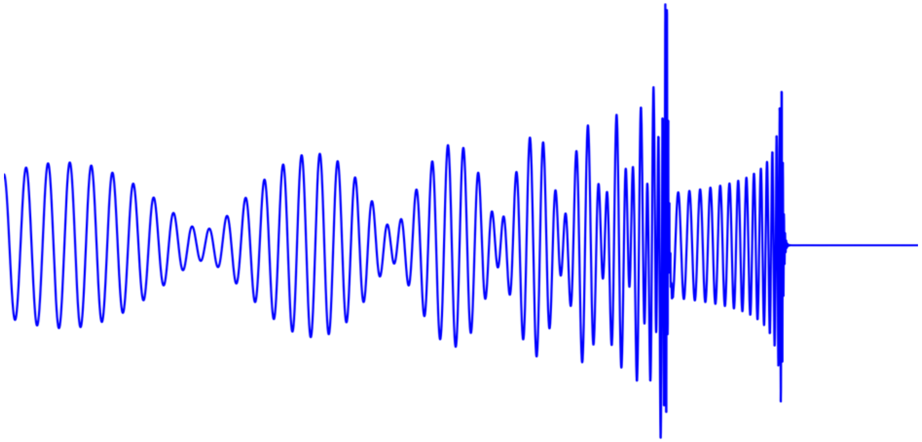

NASA/ESA, Walter Jaffe/Leiden Observatory, Holland Ford/JHU/STScI

Binaries in AGN disks

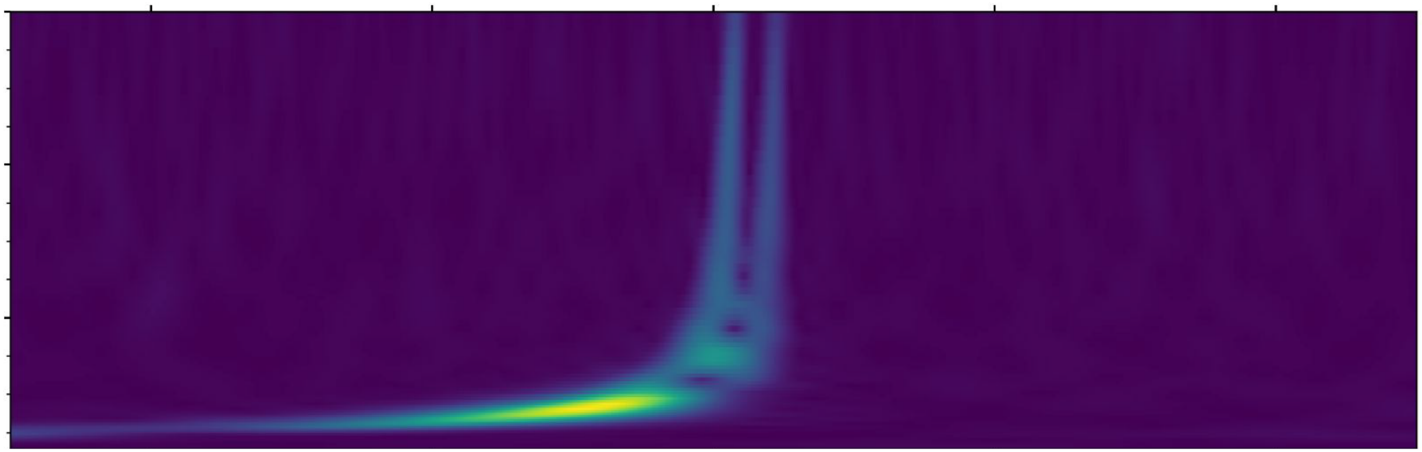
In which environments are gravitational waves «self-lensed» by astrophysical objects inside them?

Can we detect the gravitational lensing effect from each environment?

In collaboration with Mark Gieles and Jordi Miralda-Escudé



Thank you for your attention!



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