

Interference signatures on gravitational wave signals by compact gravitational lenses

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*Seminar at CFT PAS
Warsaw, 6th May 2026*



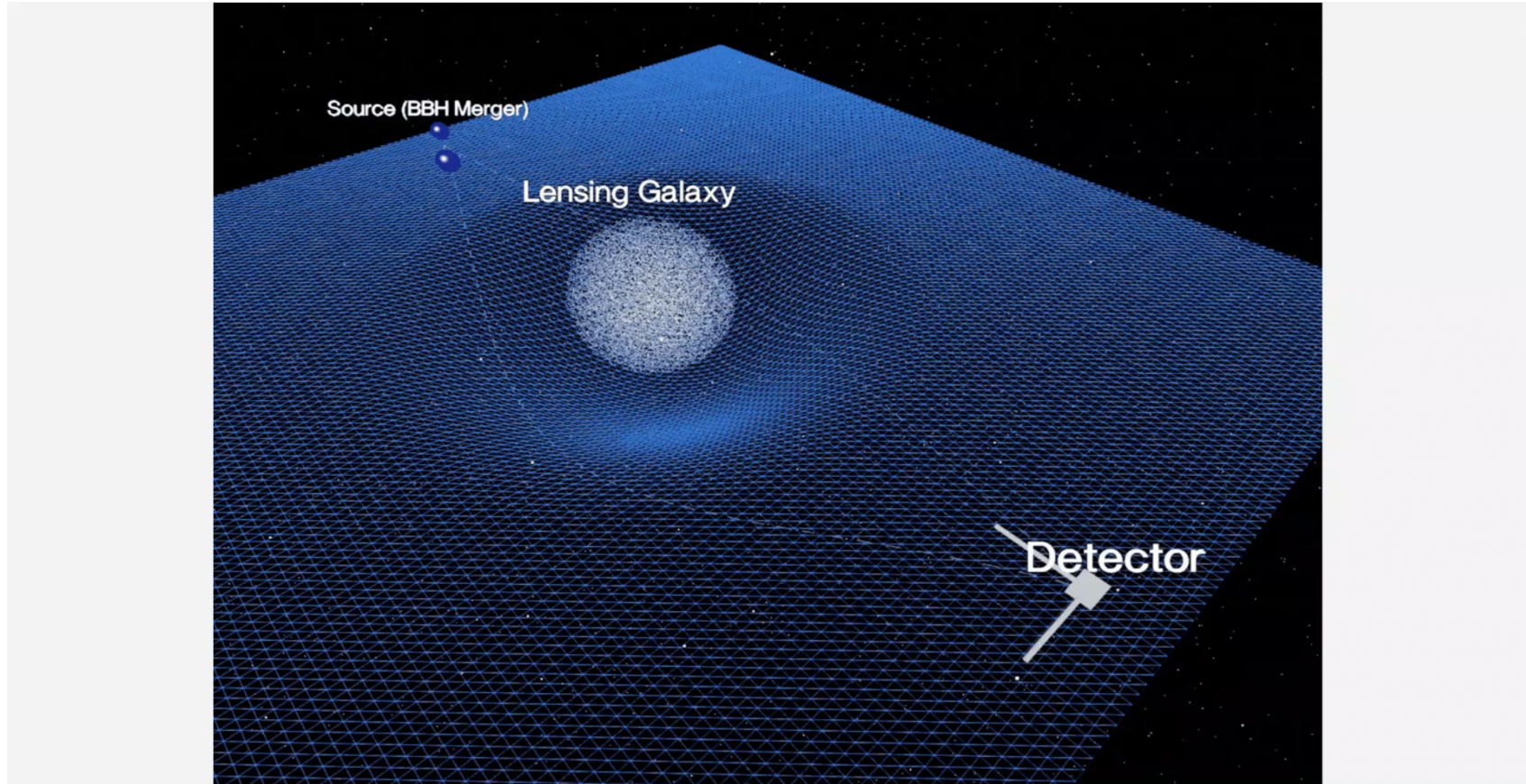
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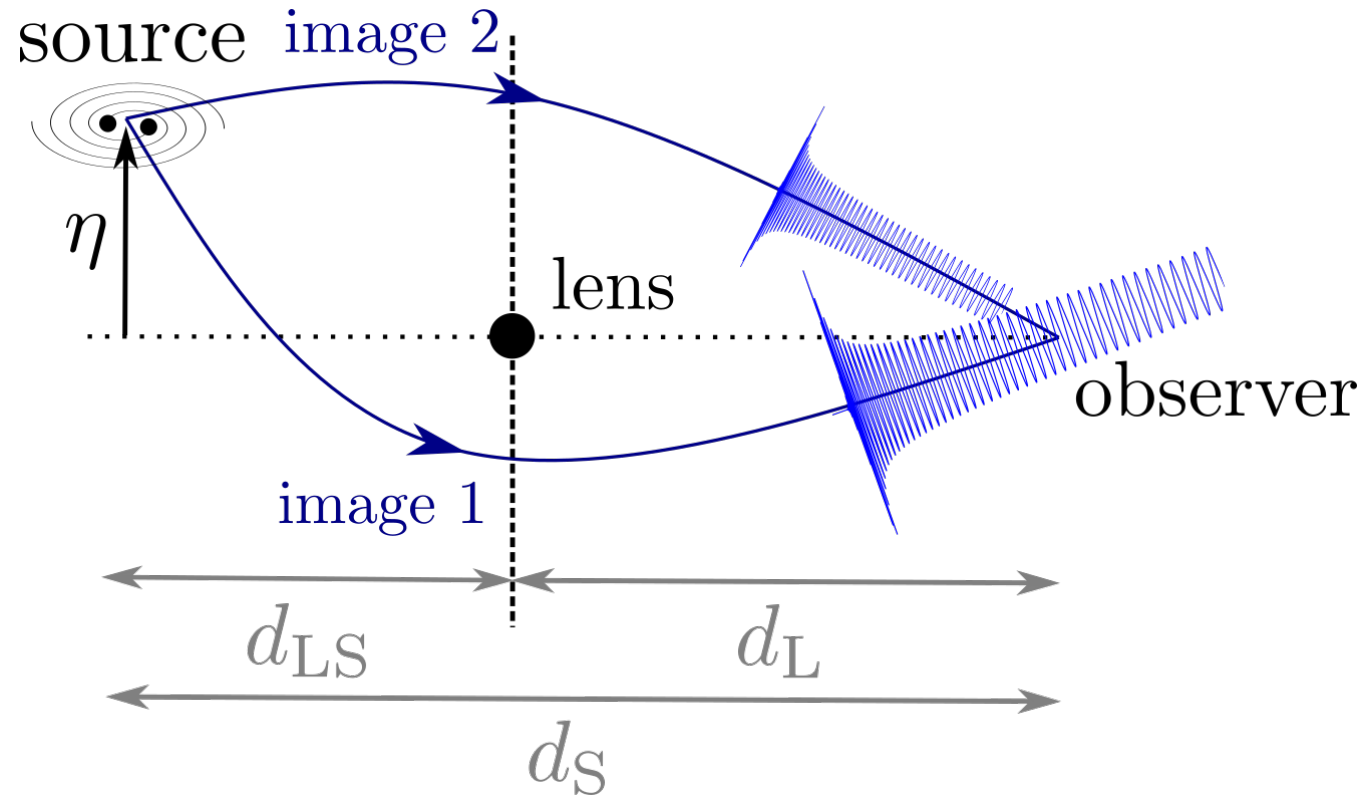


Gravitational lensing of gravitational waves



Credit: Hemantakumar Phurailatpam (CUHK)

Gravitational lensing of gravitational waves



Adapted from Ubach and Espuny (2024)

Outline

1. Why do wave effects (diffraction, interference) appear in gravitational waves (GWs)
2. Validity of the Geometrical Optics (GO) approximation
3. How do wave effects look like in GW events
4. Detectability of wave effects in GWs
5. Self-lensing in astrophysical environments
6. Self-lensing of a moving source

1. Why do wave effects appear?

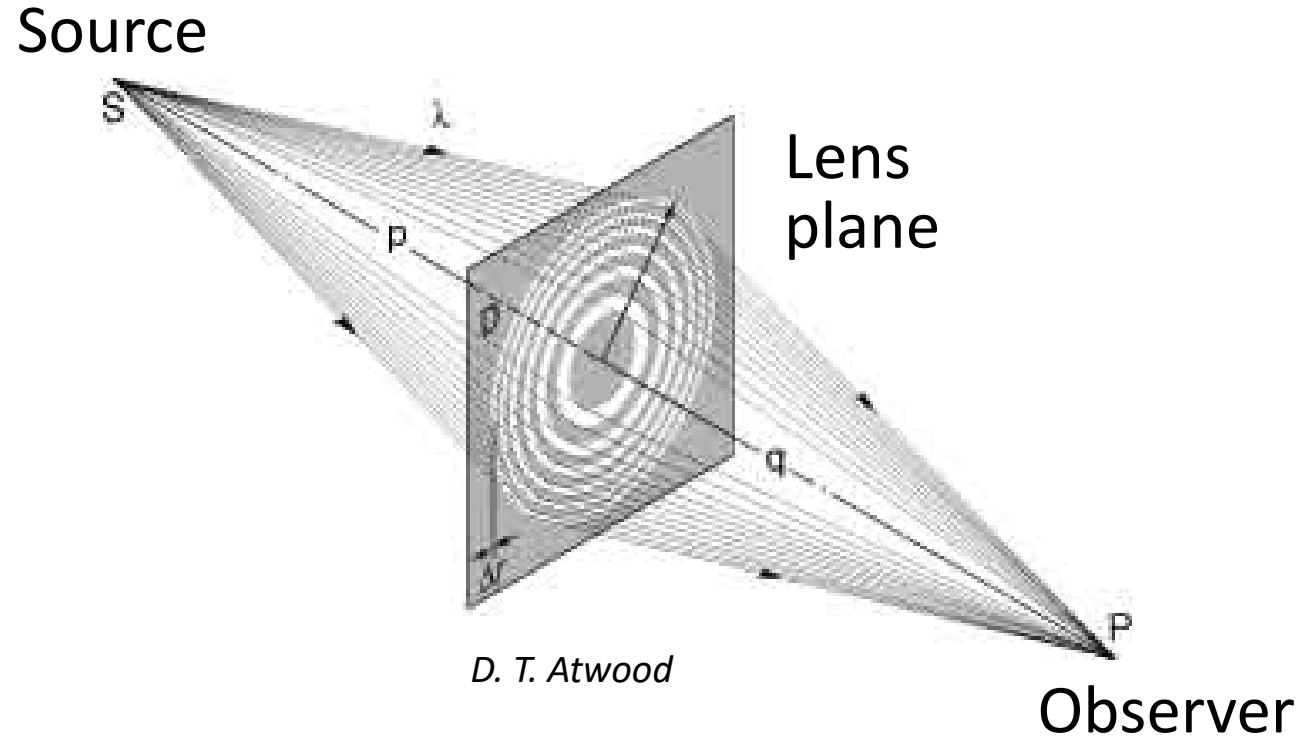
Wave effects appear as:

- Diffraction (around the lens)
- Interference (between emerging images)

Gravitational waves have:

- Coherent emission (Thorne, 1994)
- Long wavelength: $\lambda \sim R_S \rightarrow$ wave effects more common

1. Why do wave effects appear?



1. Why do wave effects appear?

Scalar field at the observer:

$$\begin{aligned} \tilde{\varphi}(P) &= \frac{A}{i\lambda} \iint_{\text{Lens}} \frac{1}{rs} e^{i[k(r+s)+\psi_L]} dS \\ &= -i\nu \frac{A}{d_S} e^{ik(r_0+s_0)} \iint_{\text{Lens}} \underbrace{e^{2\pi i \nu \tau(\mathbf{x},\mathbf{y})}}_{\Phi(\mathbf{x})} d^2\mathbf{x} \end{aligned}$$

Parameters: $\tau(y), \nu$:

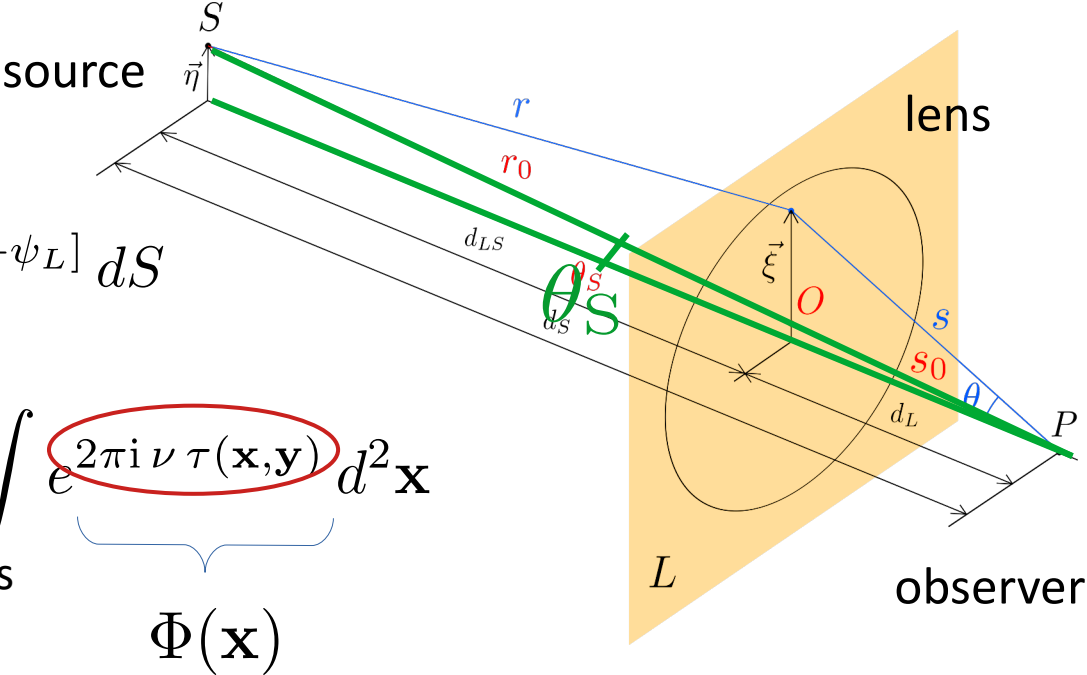
$$\tau(\mathbf{x}, \mathbf{y}) = \frac{1}{2}(\mathbf{x} - \mathbf{y})^2 - \psi(\mathbf{x}) + \psi_0 \longrightarrow \tau_{21}(y) = 2y + \frac{1}{12}y^3 + O(y^5)$$

$\nu = \frac{2R_S}{\lambda}$

Schwarzschild radius of the lens

Wavelength of gravitational waves

$R_S = \frac{2GM}{c^2}$



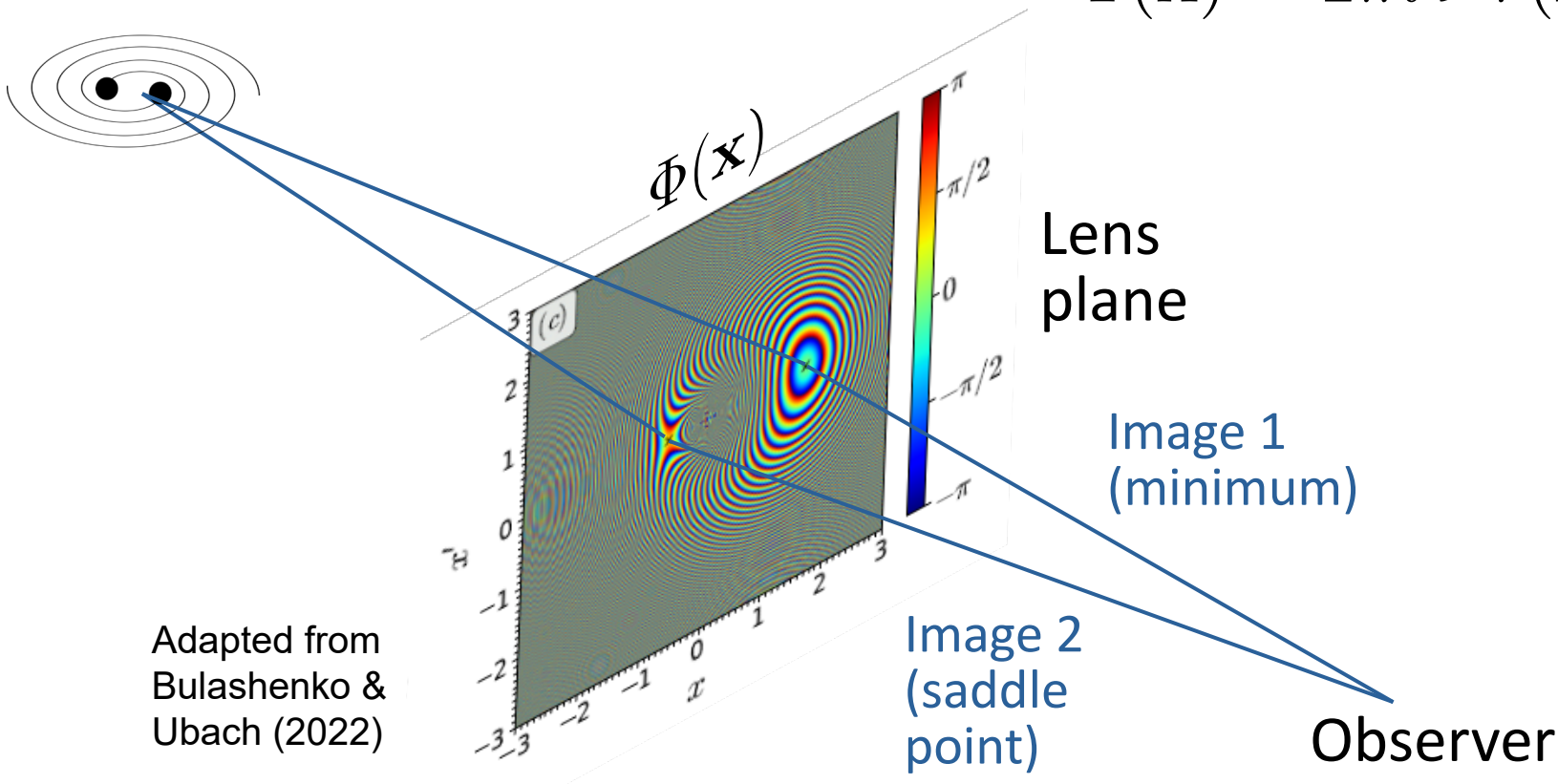
Point mass lens model

$\nu = \frac{2R_S}{c} f$

$y = \frac{\theta_S}{\theta_E}$

1. Why do wave effects appear?

$$\Phi(\mathbf{x}) = 2\pi i \nu \tau(\mathbf{x}, \mathbf{y})$$



Adapted from
Bulashenko &
Ubach (2022)

$$\tilde{\varphi}(P) \propto \sum_j |\mu_j|^{1/2} e^{i(2\pi\nu \tau(\mathbf{x}_j, \mathbf{y}) - n_j \pi/2)}$$

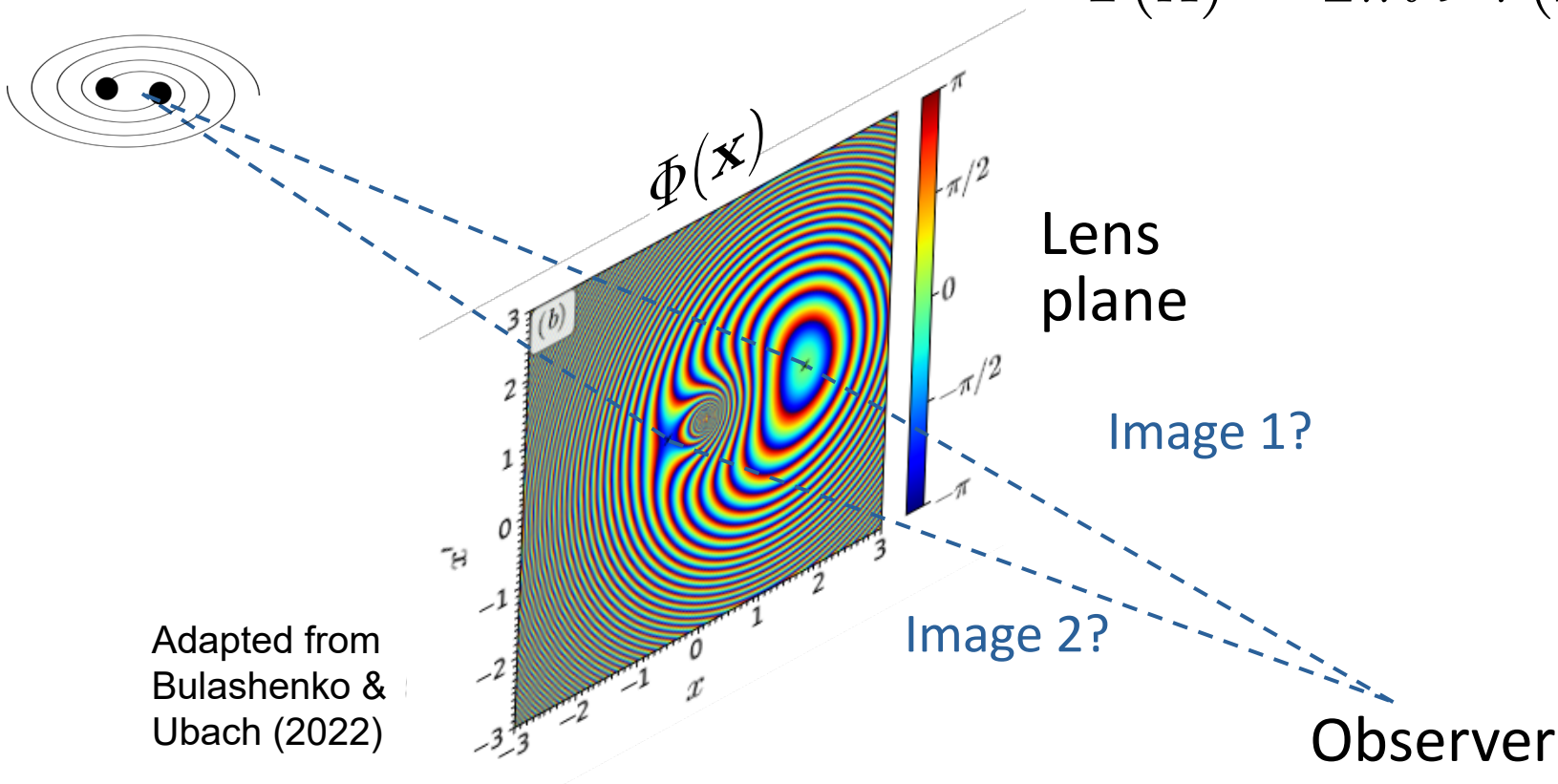
$\nu = 10$
 $(\nu \gg 1)$
 $y = 1$

Geometric Optics approximation
(Stationary Phase Approximation of the
Fresnel-Kirchhoff integral)

Point mass lens model

1. Why do wave effects appear?

$$\Phi(\mathbf{x}) = 2\pi i \nu \tau(\mathbf{x}, \mathbf{y})$$



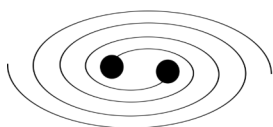
Adapted from
Bulashenko &
Ubach (2022)

$$\nu = 4$$

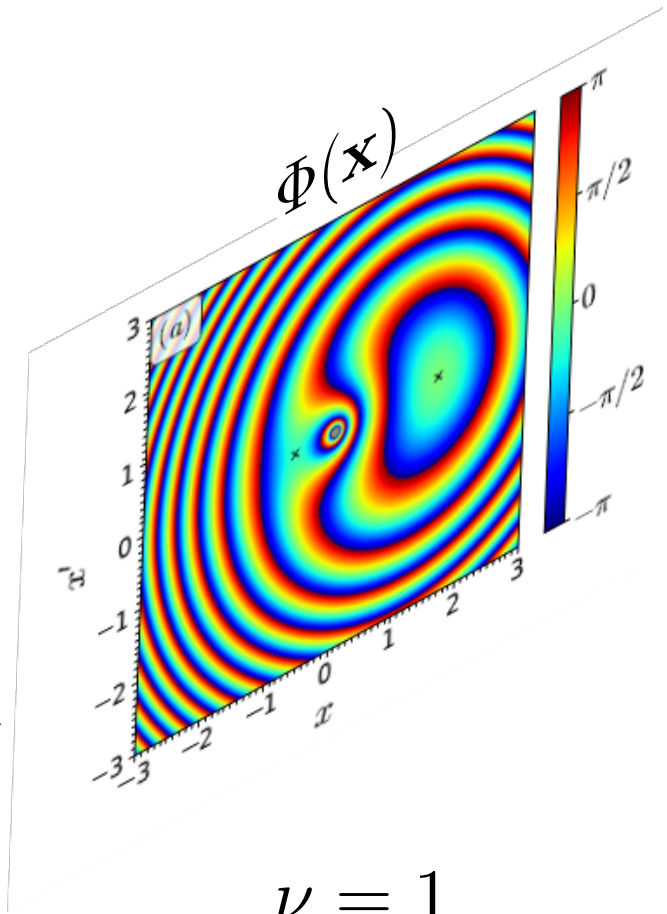
$$y = 1$$

Point mass lens model

1. Why do wave effects appear?



$$\Phi(\mathbf{x}) = 2\pi i \nu \tau(\mathbf{x}, \mathbf{y})$$



Lens plane

Observer

Adapted from
Bulashenko &
Ubach (2022)

$$\nu = 1$$

$$y = 1$$

Point mass lens model

1. Why do wave effects appear?

$$\text{Transmission factor: } F = \frac{\tilde{\varphi}(P)}{\tilde{\varphi}_0(P)} \quad \tilde{\varphi}(P) = -i\nu \frac{A}{d_S} e^{ik(r_0+s_0)} \iint e^{2\pi i \nu \tau(\mathbf{x}, \mathbf{y})} d^2\mathbf{x}$$

$$F = -\mathrm{i} \nu \iint e^{2\pi\mathrm{i} \nu \tau(\mathbf{x}, \mathbf{y})} d^2 \mathbf{x} \quad \tau(\mathbf{x}, \mathbf{y}) = \frac{1}{2}(\mathbf{x} - \mathbf{y})^2 - \psi(\mathbf{x}) + \psi_0$$

analytical solution
to Fresnel-Kirchhoff integral
(W. Gordon, 1928)

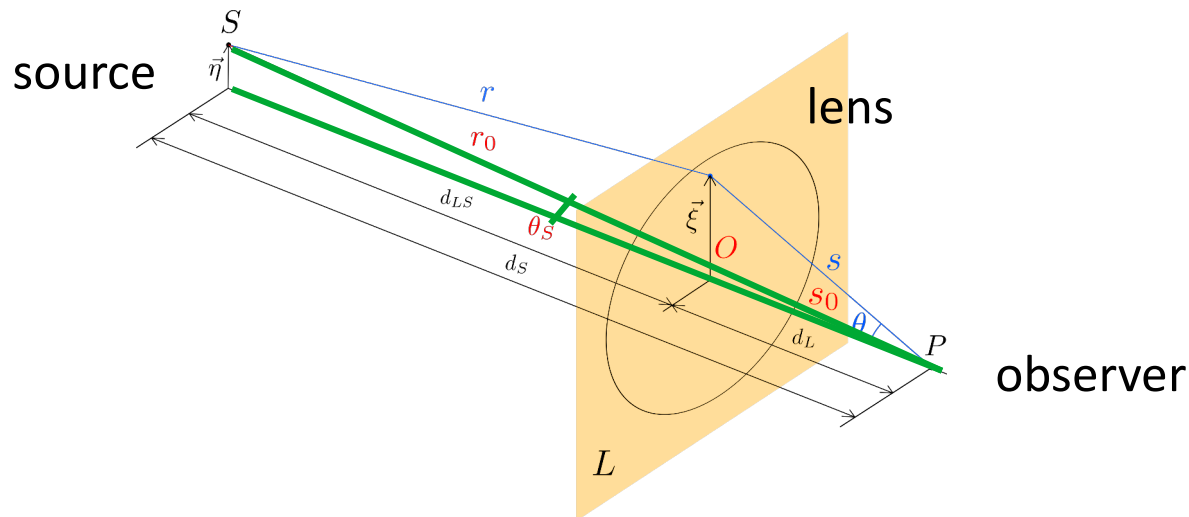
$$\downarrow$$

$$\psi(\mathbf{x}) = \ln |\mathbf{x}|$$

$$F(\textcolor{violet}{\nu}, \textcolor{green}{y}) = e^{\frac{1}{2}\pi^2\nu} e^{\mathrm{i}\pi\nu \ln(\pi\nu)} \Gamma(1 - \mathrm{i}\pi\nu) {}_1F_1(\mathrm{i}\pi\nu; 1; \mathrm{i}\pi\nu y^2)$$

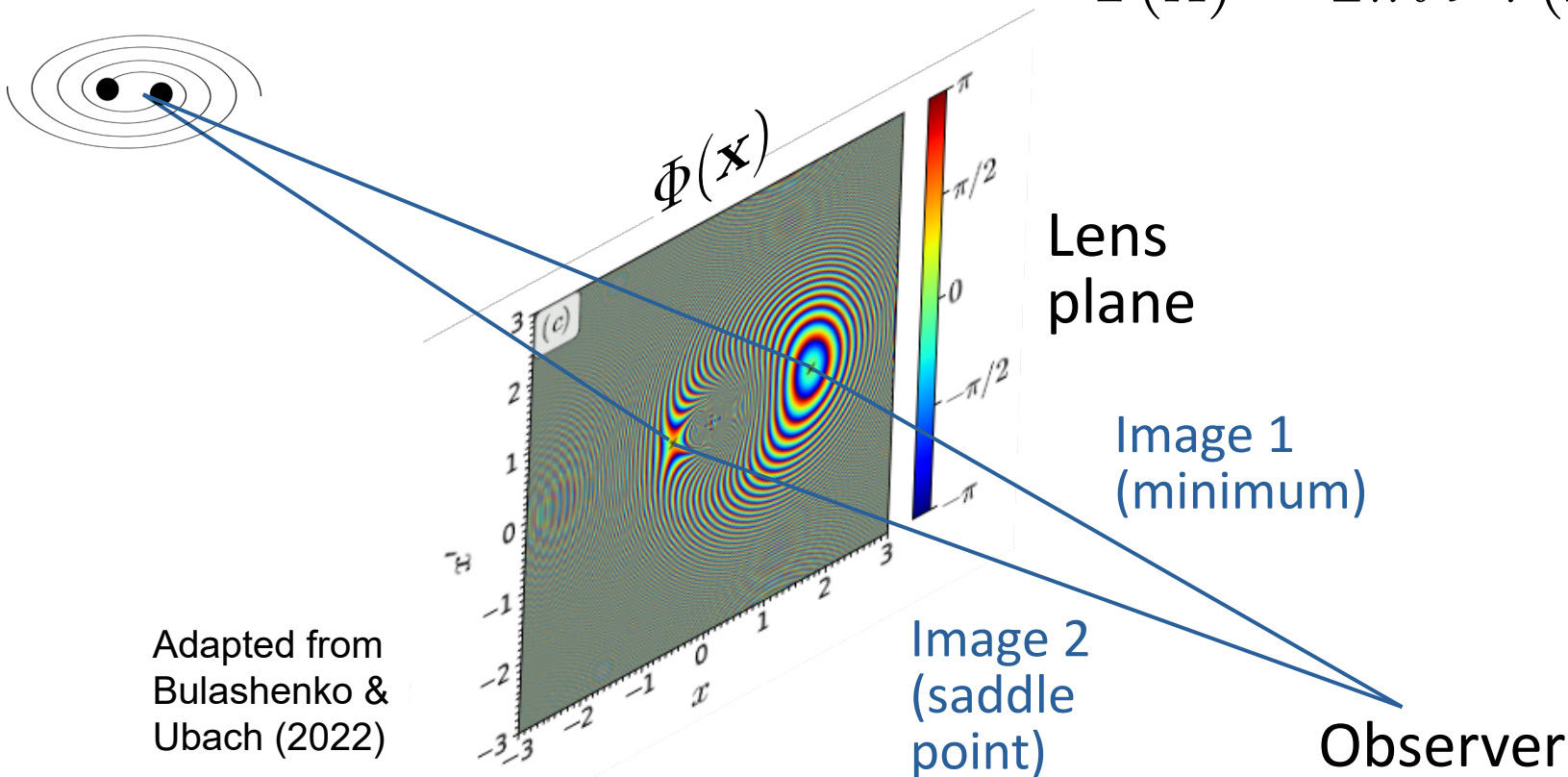
Point mass lens model

$$\nu = \frac{2R_S}{\lambda} \quad y = \frac{\theta_S}{\theta_E}$$



1. Why do wave effects appear?

$$\Phi(\mathbf{x}) = 2\pi i \nu \tau(\mathbf{x}, \mathbf{y})$$



Adapted from
Bulashenko &
Ubach (2022)

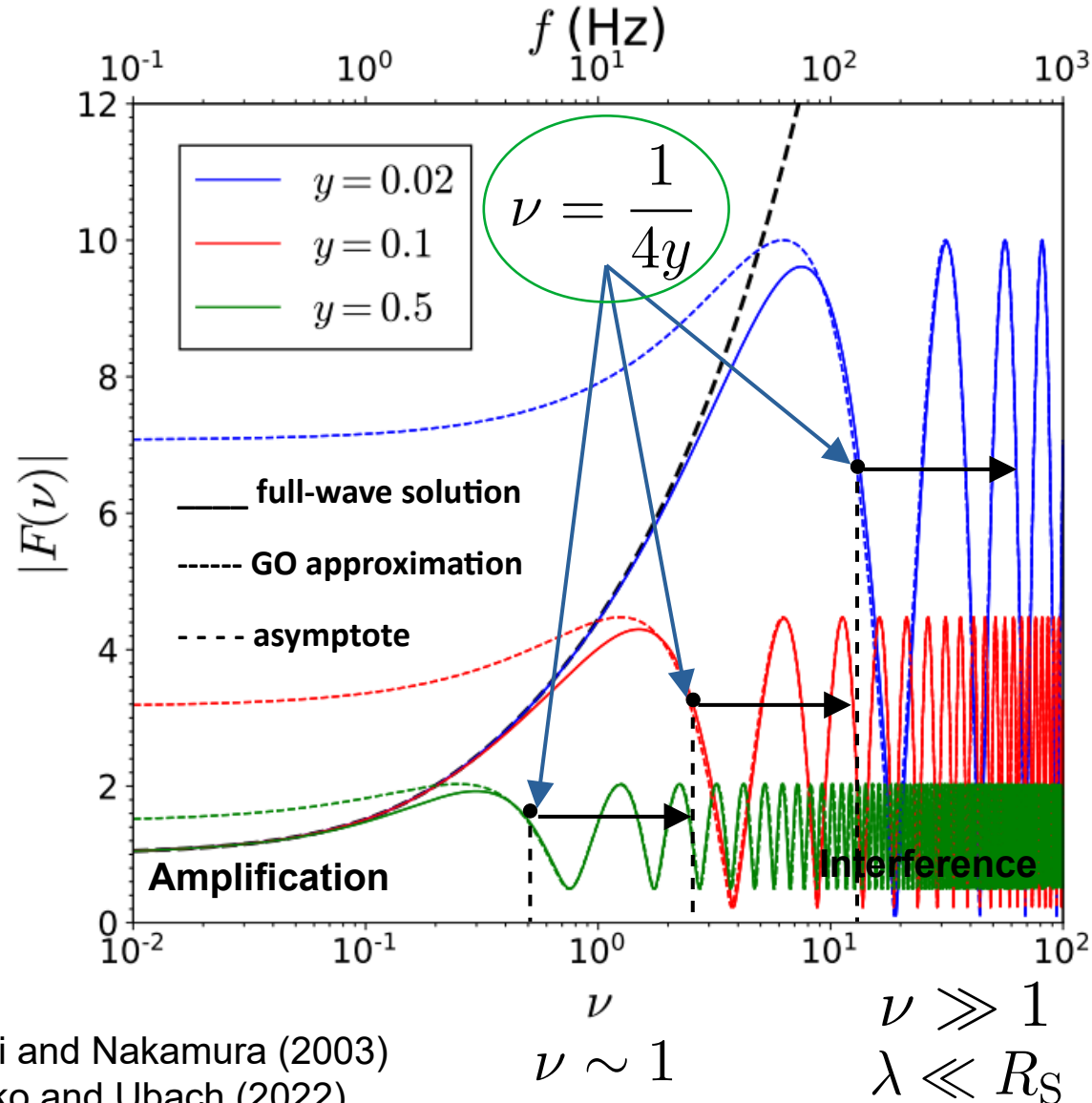
$$\tilde{\varphi}(P) \propto \sum_j |\mu_j|^{1/2} e^{i(2\pi\nu \tau(\mathbf{x}_j, \mathbf{y}) - n_j \pi/2)}$$

$\nu = 10$
 $(\nu \gg 1)$
 $y = 1$

Geometric Optics approximation
(Stationary Phase Approximation of the
Fresnel-Kirchhoff integral)

Point mass lens model

2. Validity of the GO approximation



GO approximation
valid:

~~$\nu \gg 1$~~ ?

$\nu \gtrsim \frac{1}{4y}$

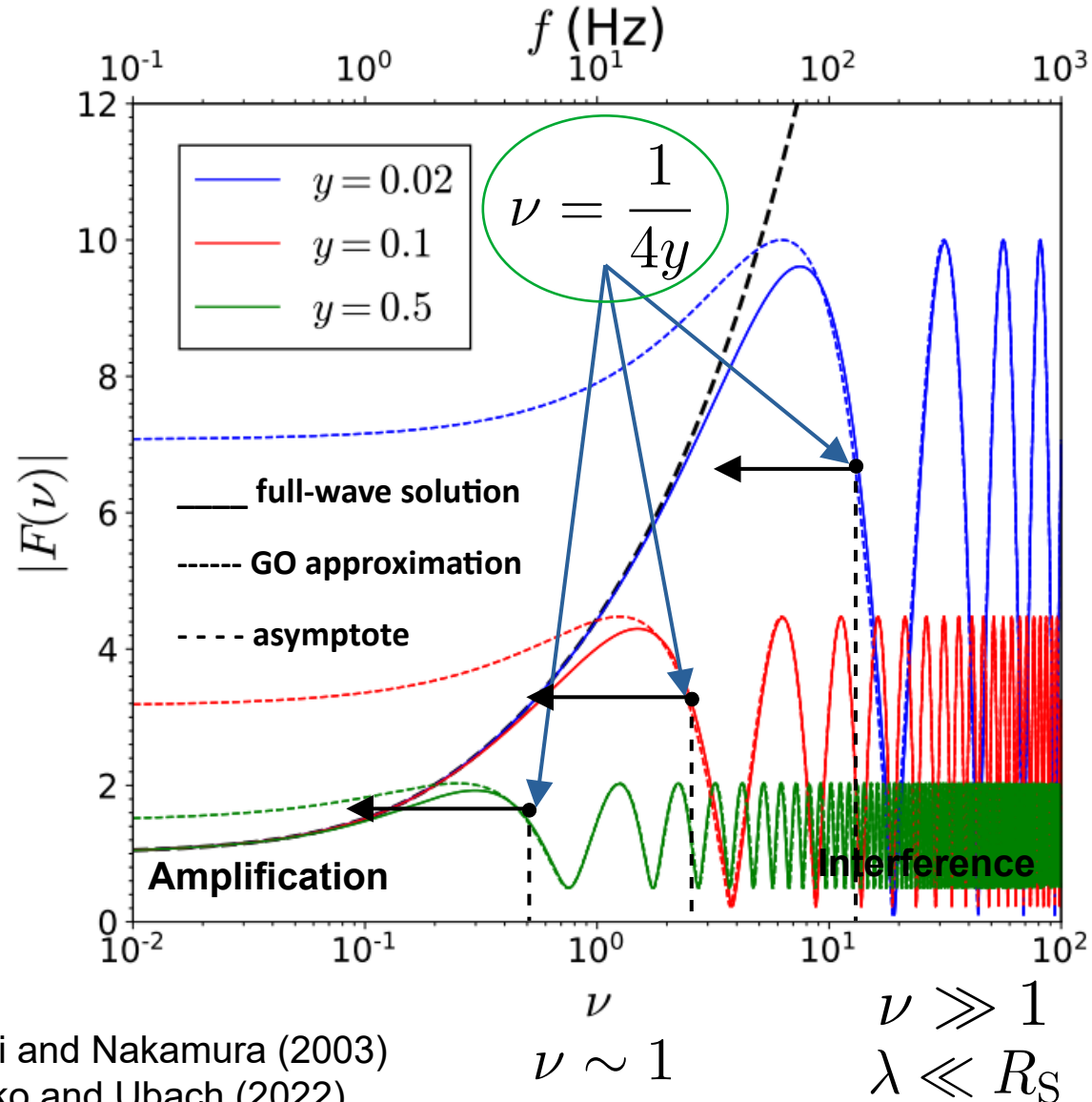
$\Downarrow$

~~$\lambda \ll R_S$~~

$\lambda \lesssim 8R_S y$

$1/f_{\text{GW}} \lesssim \tau_{21}$

2. Validity of the GO approximation



Diffraction significant:

$$\cancel{\nu \lesssim 1}$$

$$\nu \lesssim \frac{1}{4y}$$

$\Downarrow$

$$\cancel{\lambda \gtrsim R_S}$$

$$\lambda \gtrsim 8R_S y$$

$$1/f_{\text{GW}} \gtrsim \tau_{21}$$

2. Validity of the GO approximation

For our astrophysical case of interest:
Binary mergers emitting gravitational waves

$$\nu = f \frac{4G M_{\text{Lens}}}{c} \lesssim \frac{1}{4y} \quad \longrightarrow \quad \boxed{\frac{M_{\text{Lens}}}{M_{\text{Source}}} \gtrsim \frac{1}{y}}$$

$f_{\text{Ringdown}} \simeq 1.2 \times 10^4 \text{ Hz} \left(\frac{M_{\odot}}{M_{\text{Source}}} \right)$

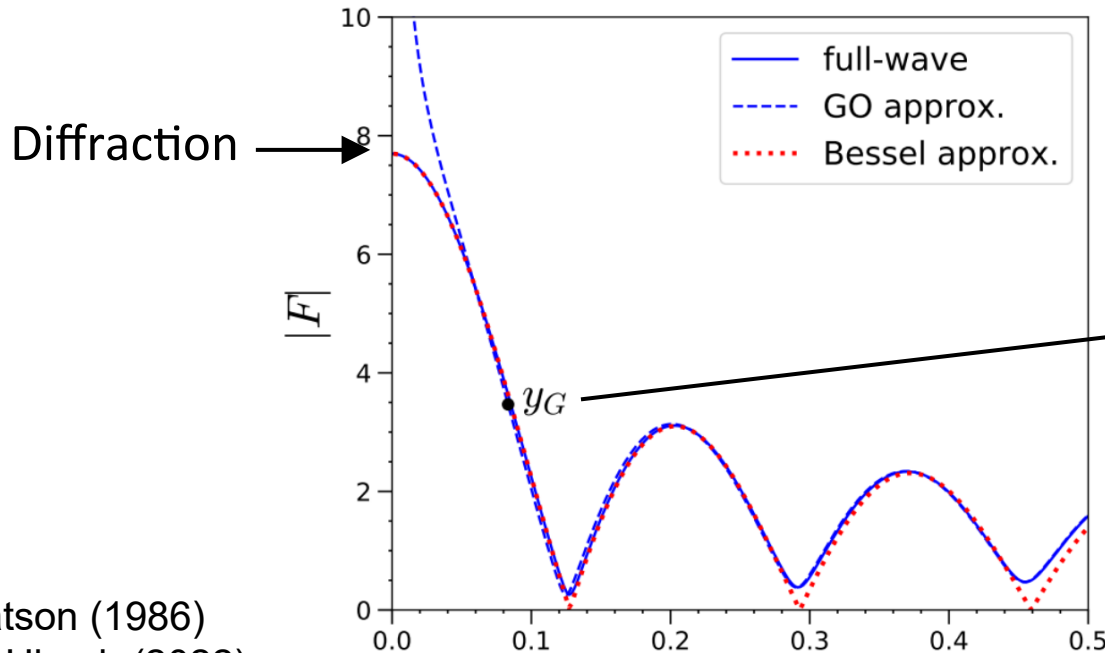
Source: compact binary merger
(Ringdown = last stage of merger)

2. Validity of the GO approximation

$$\boxed{\frac{M_{\text{Lens}}}{M_{\text{Source}}} \gtrsim \frac{1}{y}} \quad \left\{ \begin{array}{l} y \sim 1 \longrightarrow M_{\text{Lens}} \gtrsim M_{\text{Source}} \\ y \ll 1 \longrightarrow M_{\text{Lens}} \gg M_{\text{Source}} \end{array} \right.$$

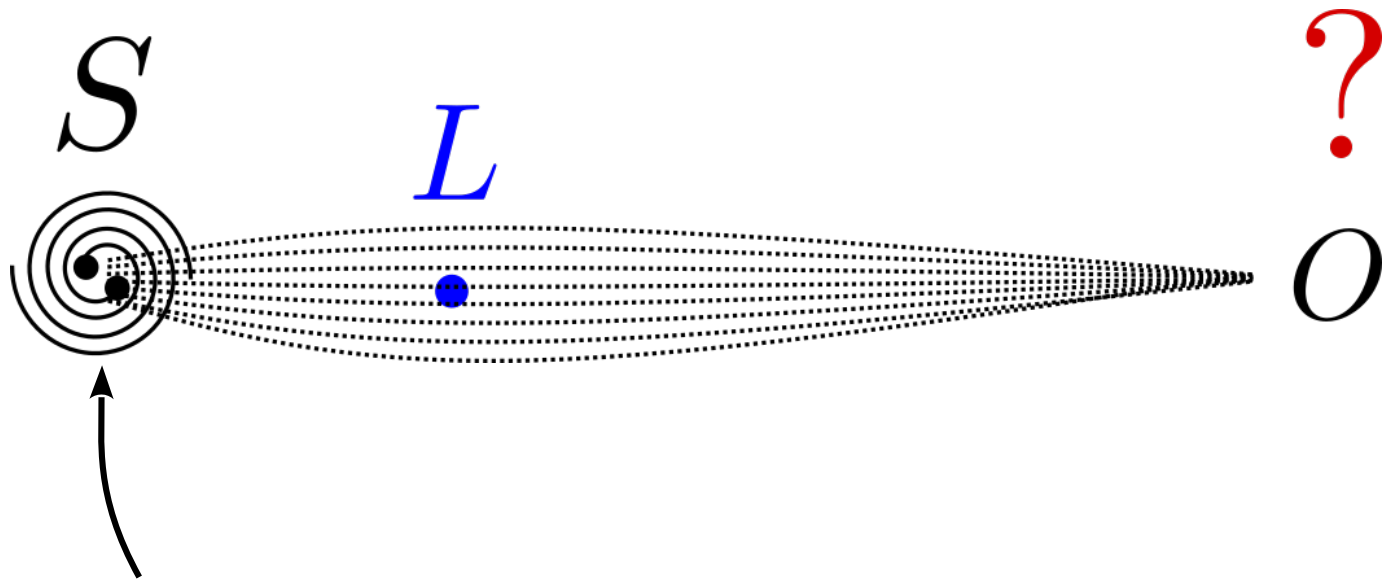
Close to caustics, wave effects are also important, even for large M_{Lens}

$$y \ll 1$$



$$y_G \sim \frac{M_{\text{Source}}}{M_{\text{Lens}}}$$

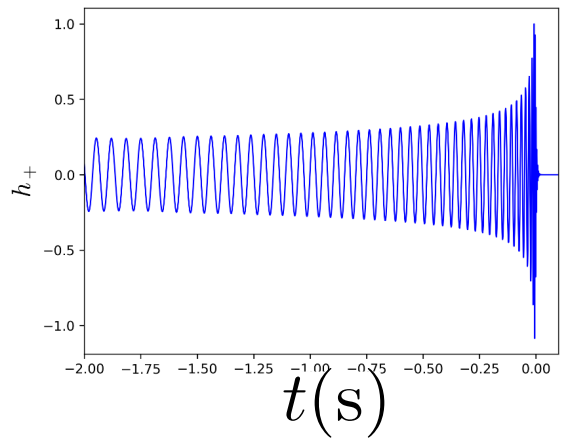
3. How do wave effects look like?



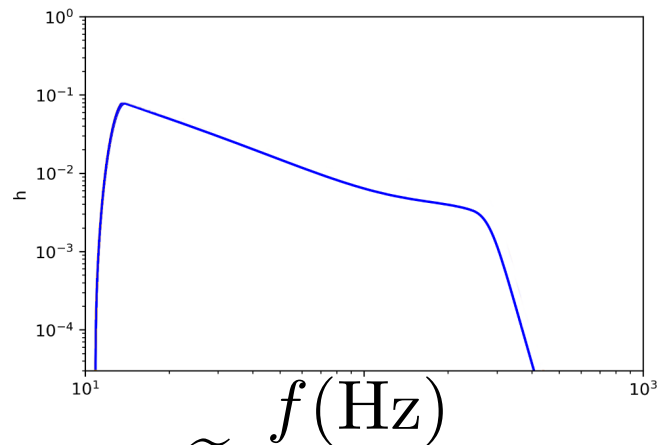
*Focus on compact binary mergers
(LIGO-Virgo-KAGRA frequencies)*

3. How do wave effects look like?

Compact binary merger



Fourier Transform



$h_{\text{unlensed}}(t)$

$\tilde{h}_{\text{unlensed}}(f)$

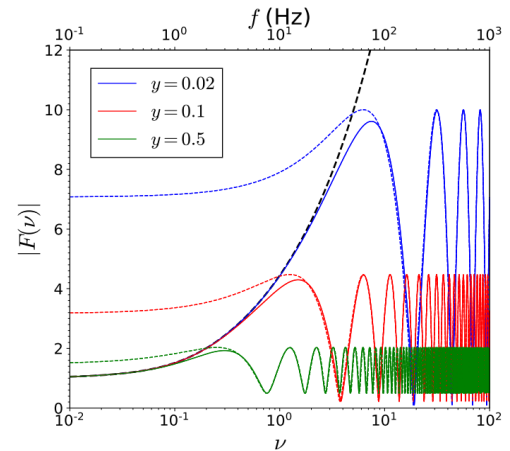
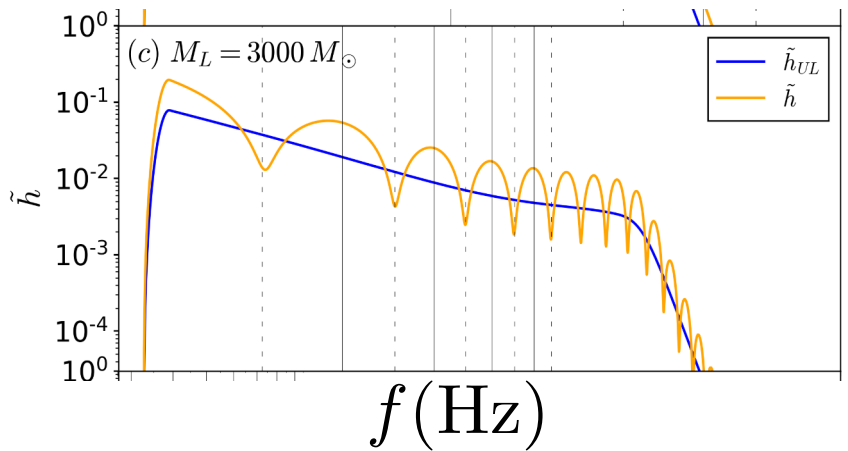
$\times$
 $|F(f)|$

Inverse
Fourier
Transform

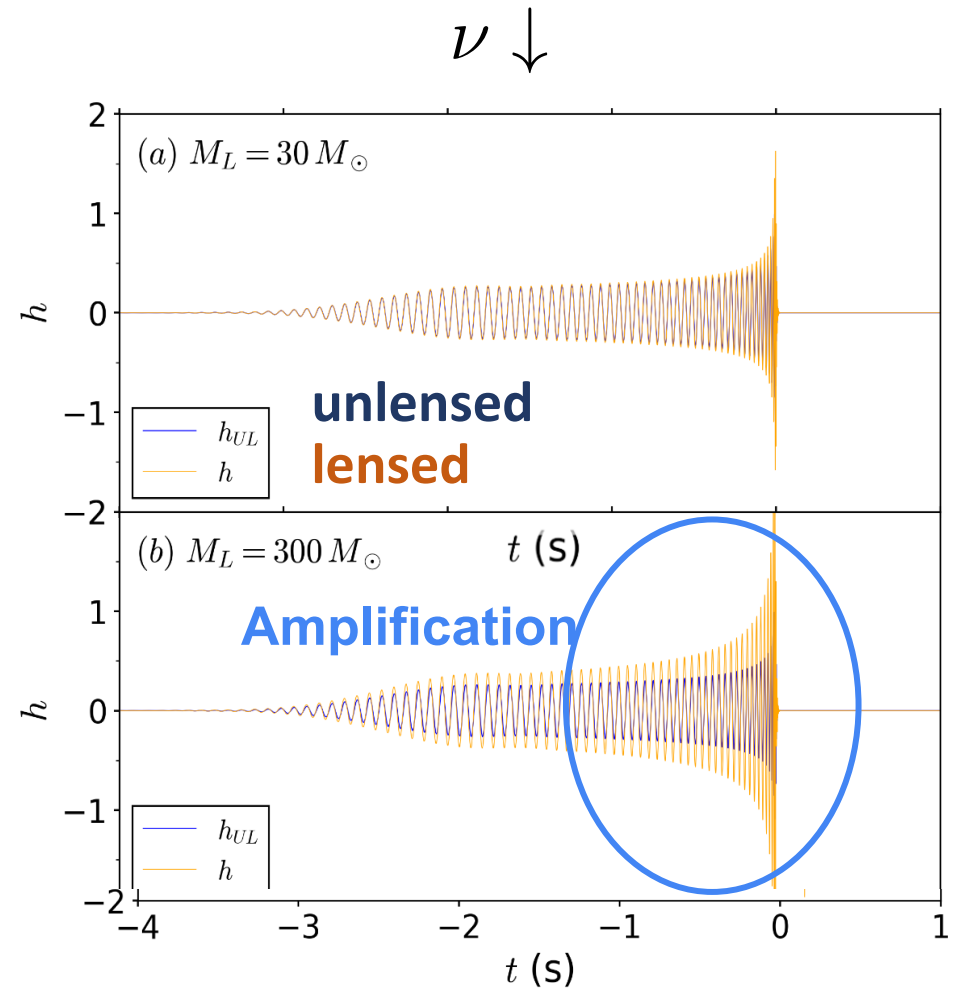
$h_{\text{lensed}}(t)$

?

$$|\tilde{h}_{\text{lensed}}(f)| = |\tilde{h}_{\text{unlensed}}(f)| \cdot |F|(f)$$

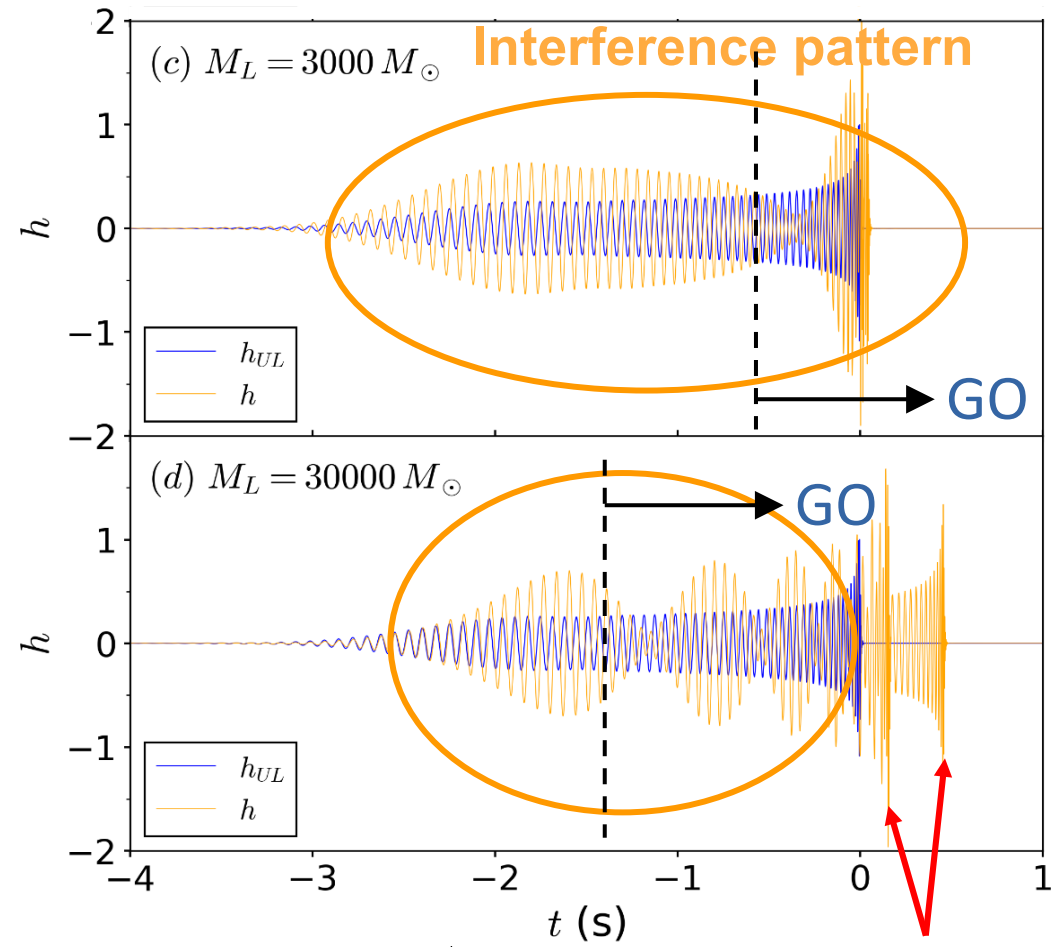


3. How do wave effects look like?



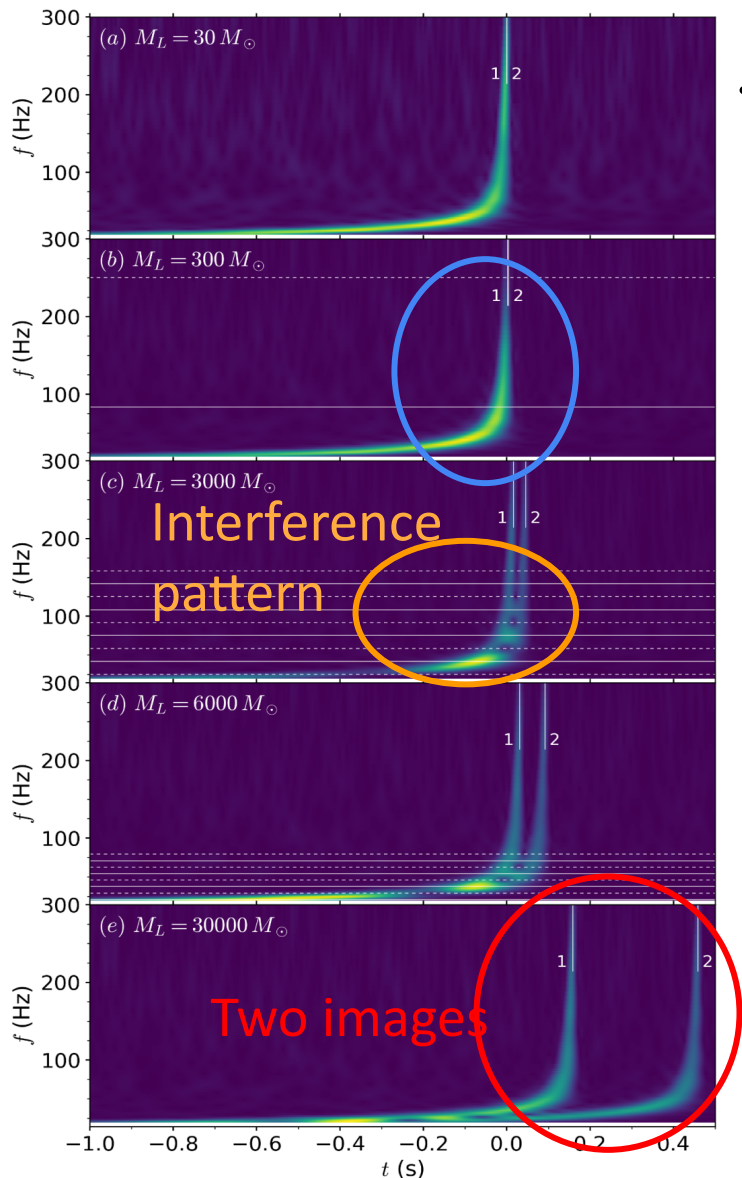
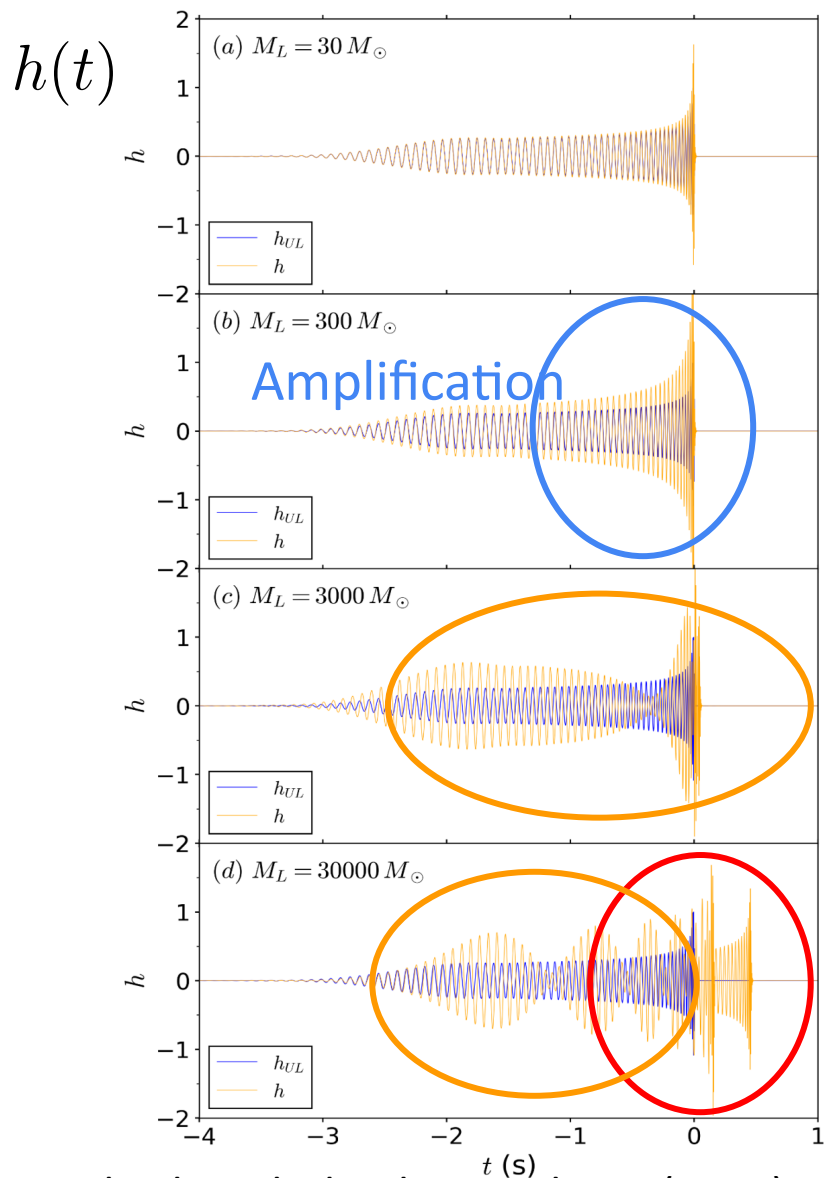
Bondarescu, Ubach, Bulashenko, Lundgren (2023)

Point mass lens model, $y = 0.25$



$\nu \uparrow$

3. How do wave effects look like?



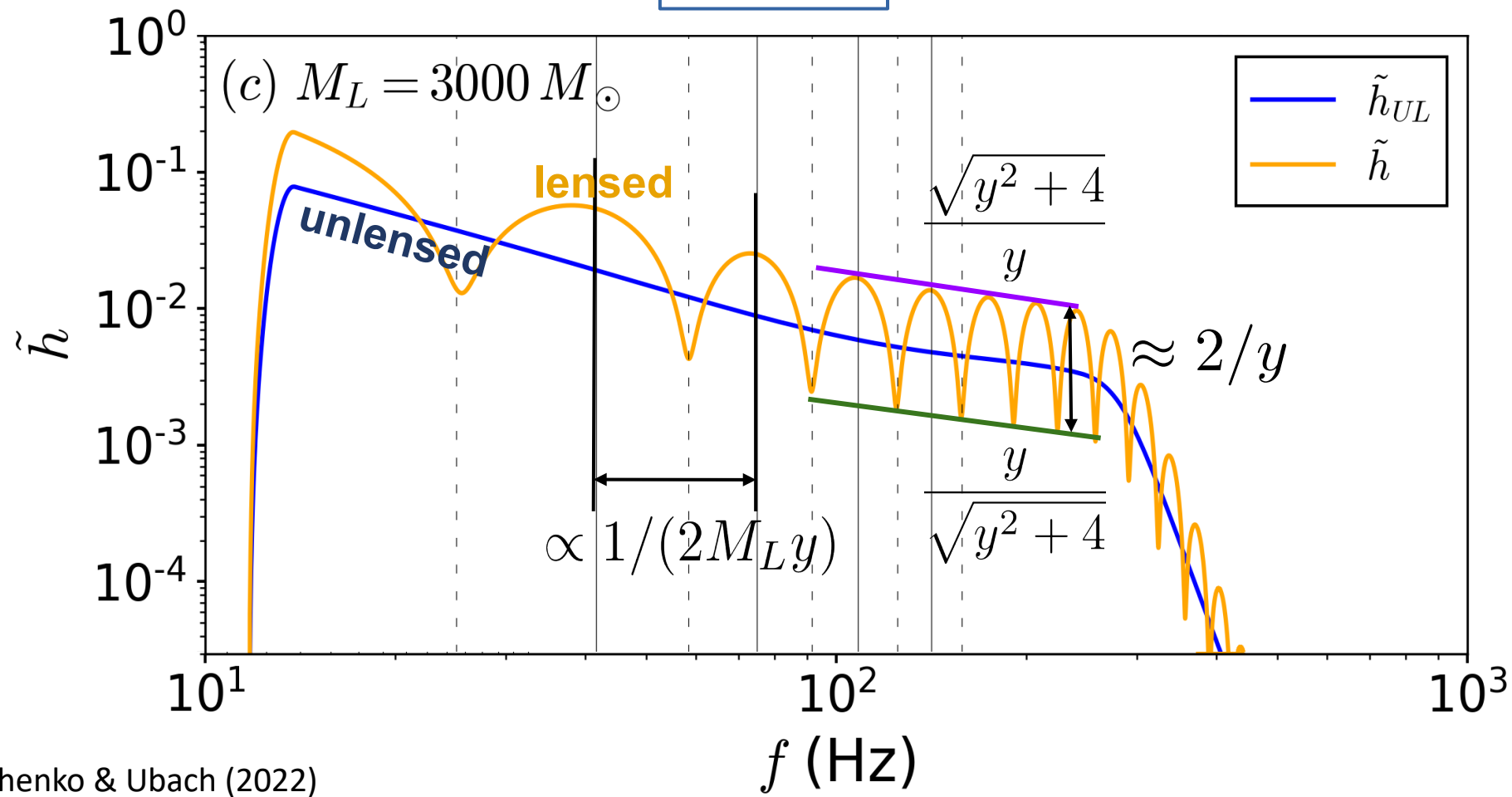
$y = 0.25$

3. How do wave effects look like?

Evenly spaced interference pattern

$y = 0.25$

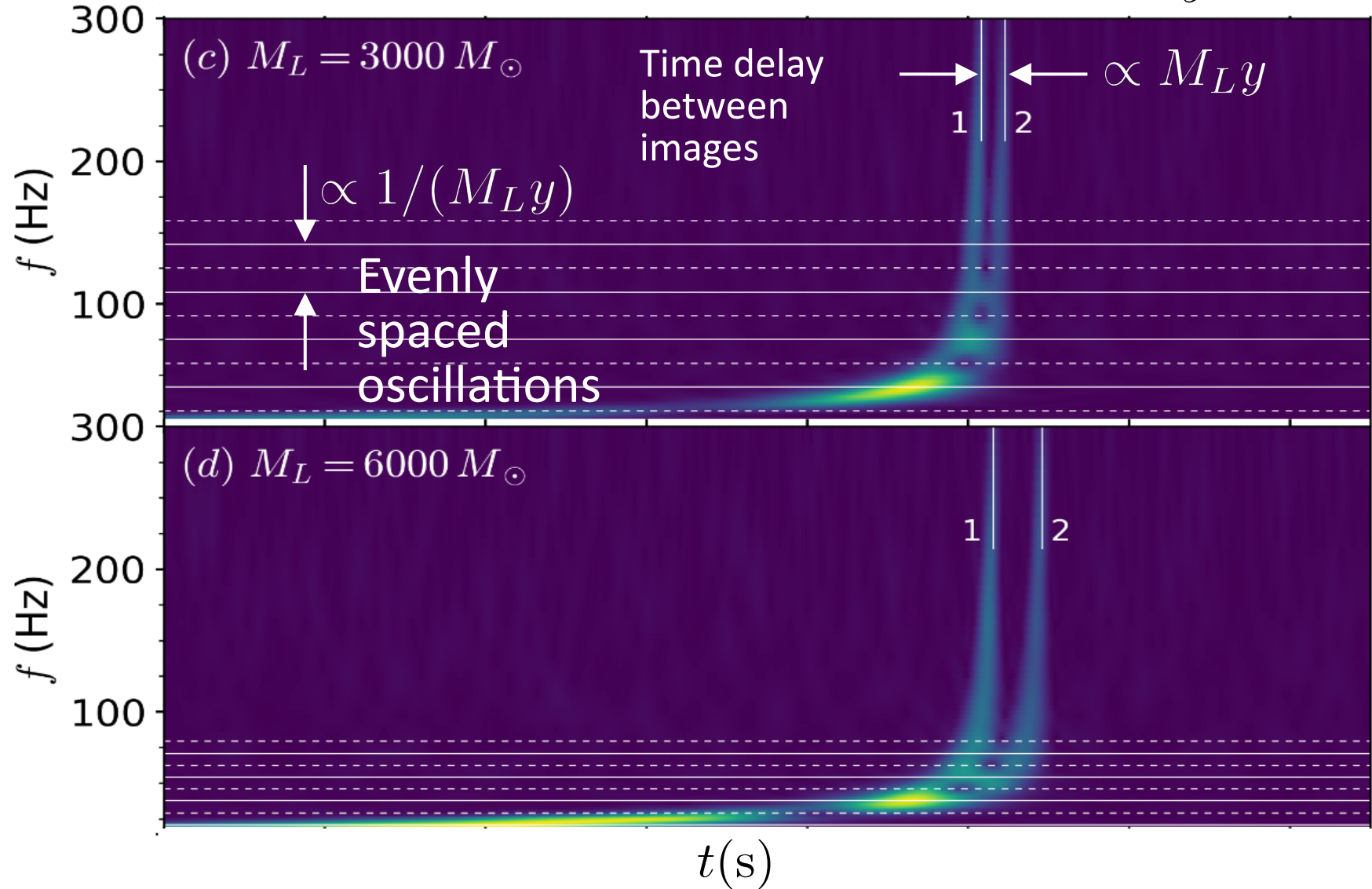
$y \lesssim 0.5$



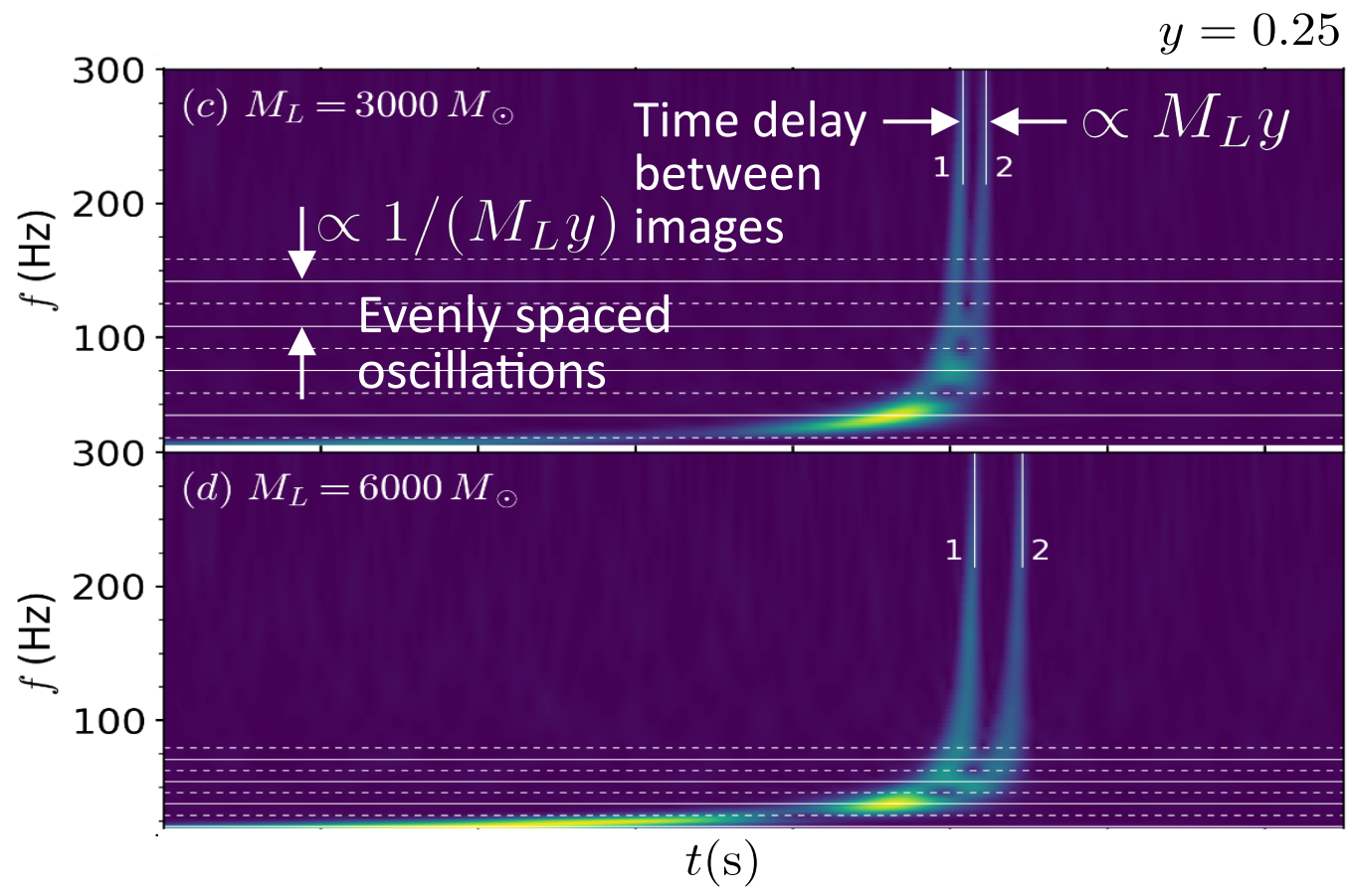
Bulashenko & Ubach (2022)
Bondaescu, Ubach, Bulashenko, Lundgren (2023)

3. How do wave effects look like?

$y = 0.25$



3. How do wave effects look like?



Time delay between images Δt

Spacing between oscillations (consecutive maxima/minima) Δf

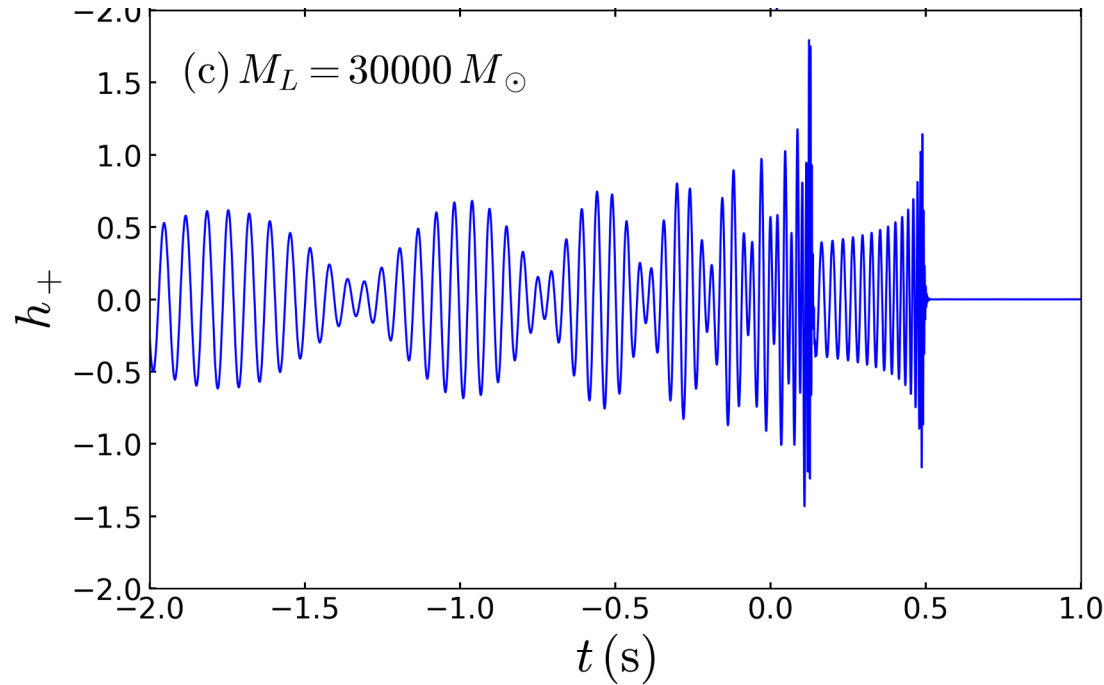
$$\Delta t \sim \frac{1}{\Delta f}$$

Signatures of microlensing:
Evenly spaced oscillations + Time-delayed images

→ To distinguish from eccentric, precessing and other frequency-modulated signals

3. How do wave effects look like?

Interfering gravitational waves (+sonification)



Ubach & Espuny, [arXiv:2407.09588](https://arxiv.org/abs/2407.09588) (2024)

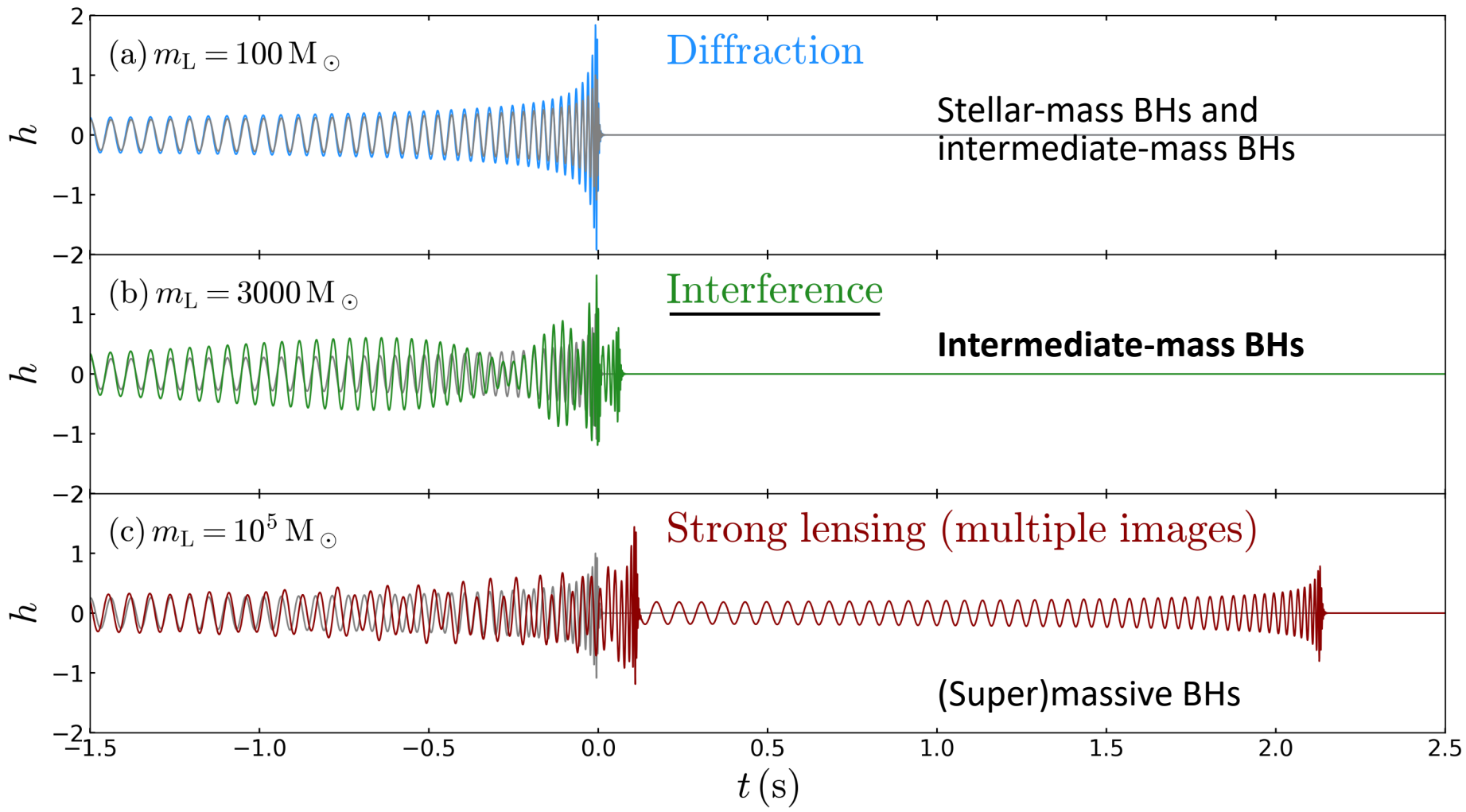
***Geometric Optics**

***Point mass lens model**

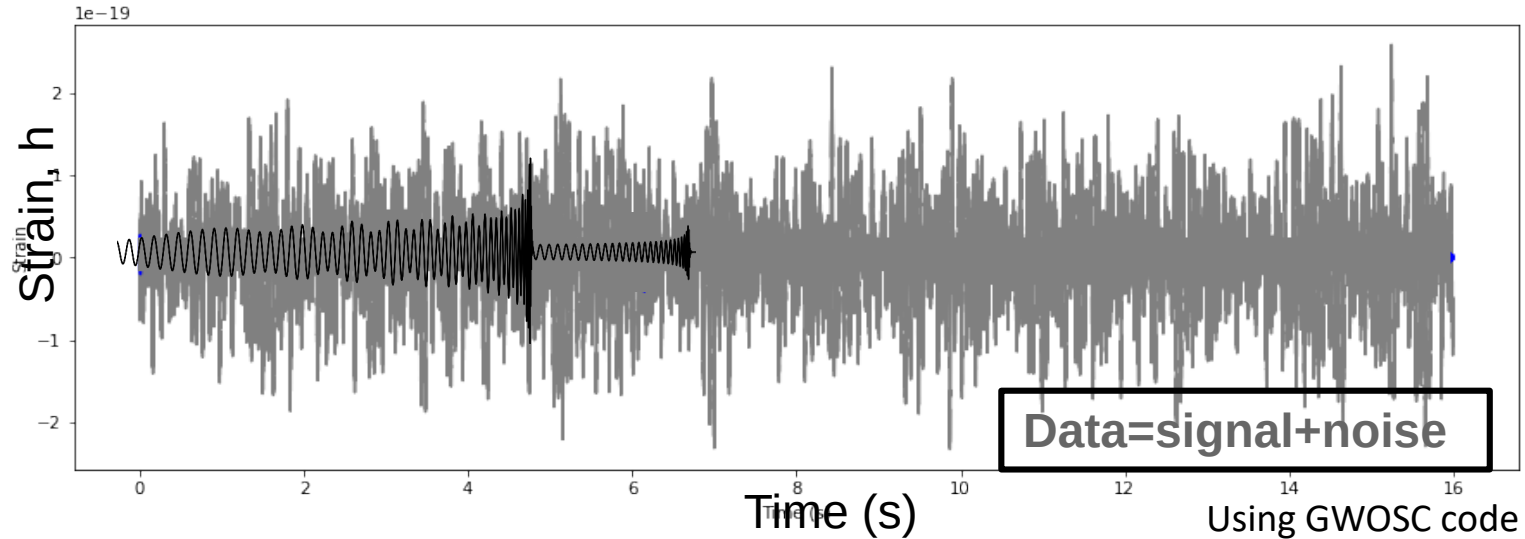
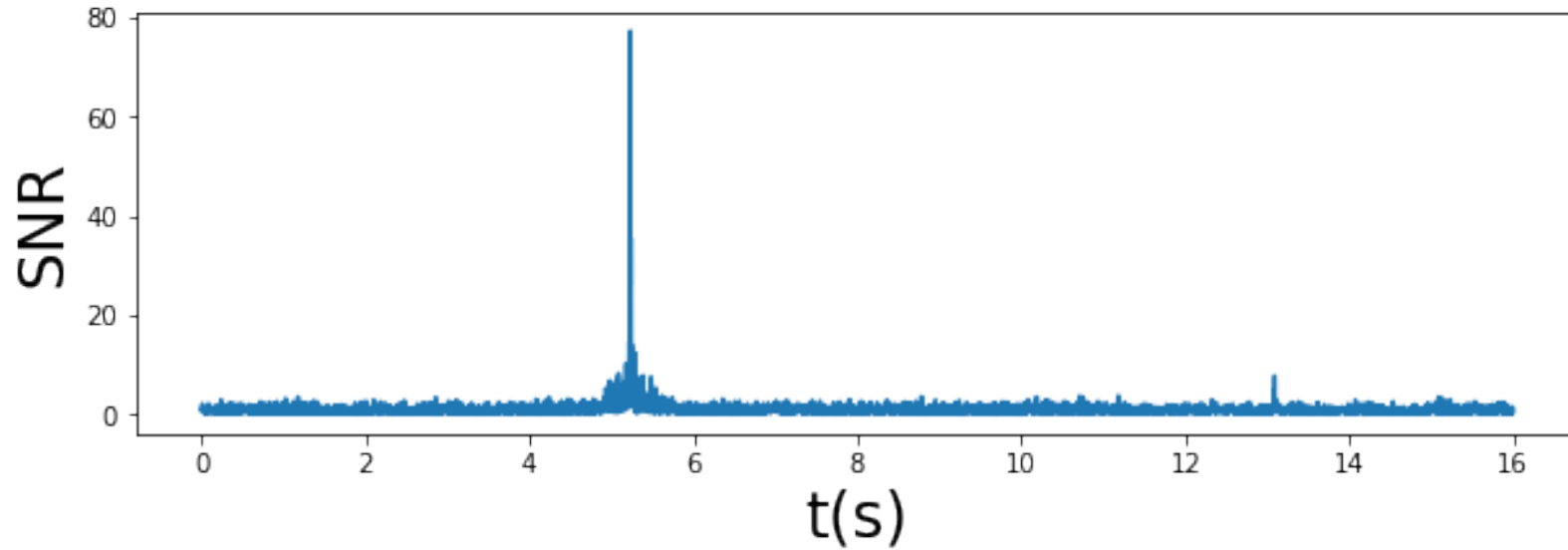
Sonifications in collaboration with Jordi Espuny:

<https://zoom3.net/sonificaciones/>

3. How do wave effects look like?



4. Detectability of wave effects

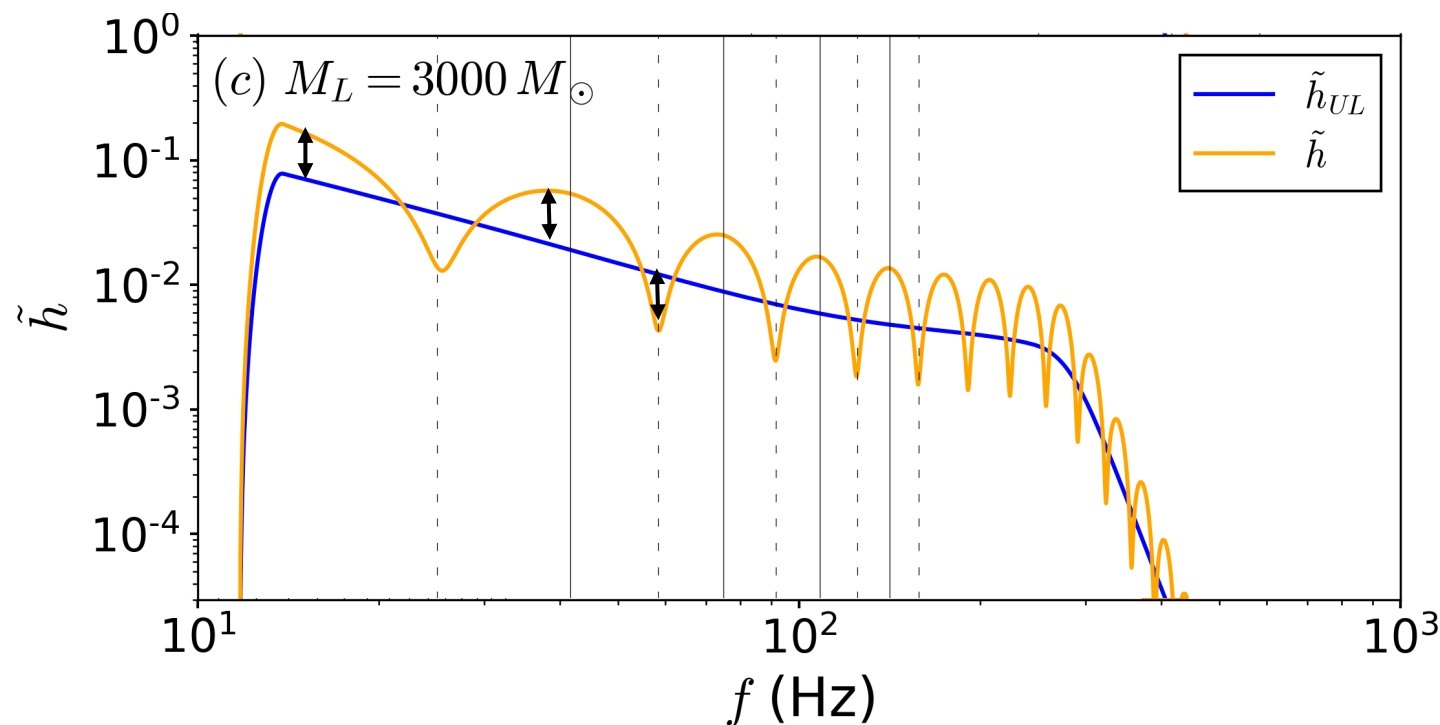


4. Detectability of wave effects

Match between templates, $\mathcal{M}$

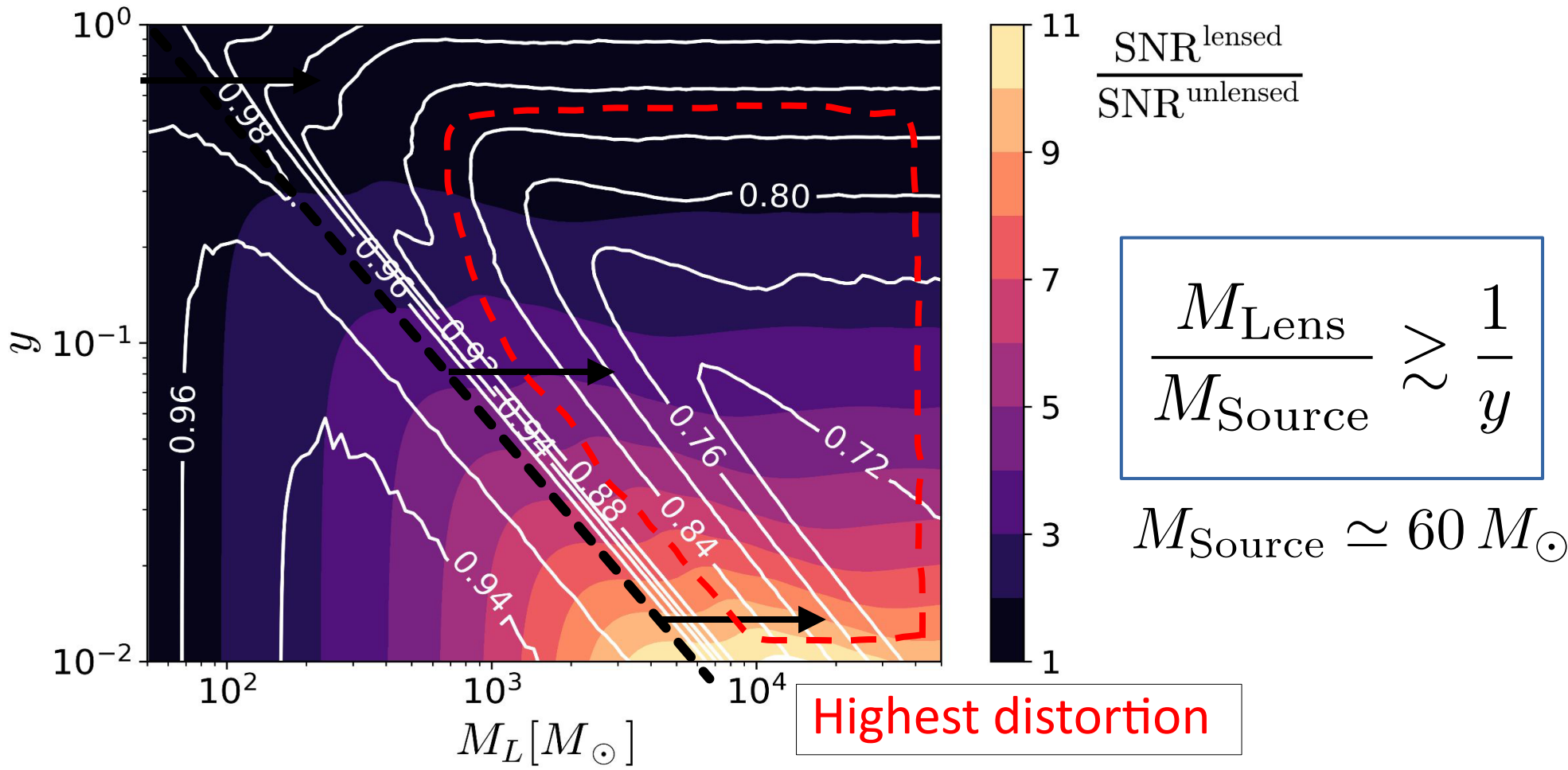
Quantifies the non-deviation on of **one template** from **another template**, integrated over frequency

$$\mathcal{M}(h, h_{UL}) = \frac{\langle f^{1/2} h_{UL}, h_{UL} \rangle}{\sqrt{\langle h_{UL}, h_{UL} \rangle \langle \sqrt{f} h_{UL}, \sqrt{f} h_{UL} \rangle}}, \quad \langle a, b \rangle = 2 \int_{f_{\min}}^{f_{\max}} \frac{\tilde{a}^*(f) \tilde{b}(f) + \tilde{a}(f) \tilde{b}^*(f)}{S_n(f)} df$$



4. Detectability of wave effects

Match determines distortion from the unlensed signal

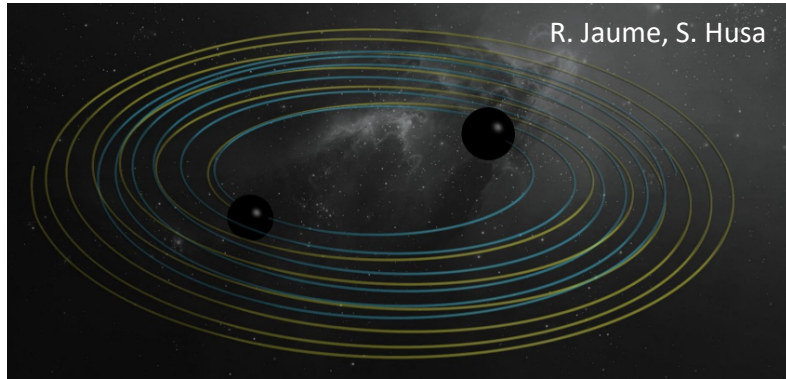


Bondarescu, Ubach, Bulashenko, Lundgren (2023)
See also Mishra+(2024)

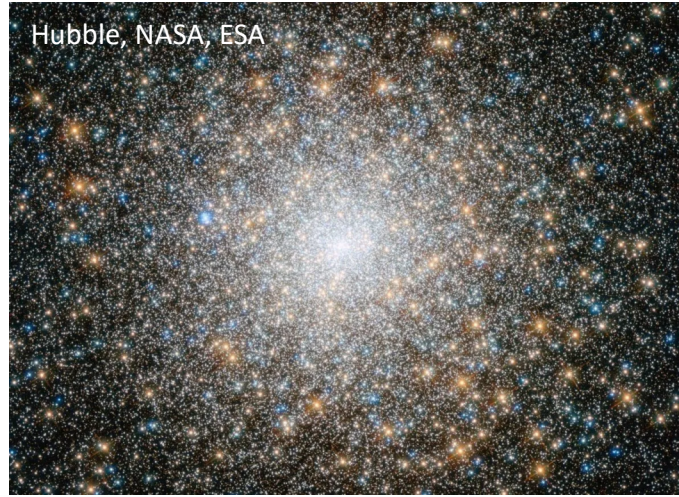
*However, see Chan+(2025)
→ detectability might decrease in wave-optics region

5. Self-lensing

Gravitational waves from different astrophysical environments:



Isolated binaries, «in the field»



Binaries in **star clusters**



Binaries in **AGN disks**

How to distinguish the environment of GW sources?

Spins, eccentricity, Doppler/phase shifts..., self-lensing

5. Self-lensing

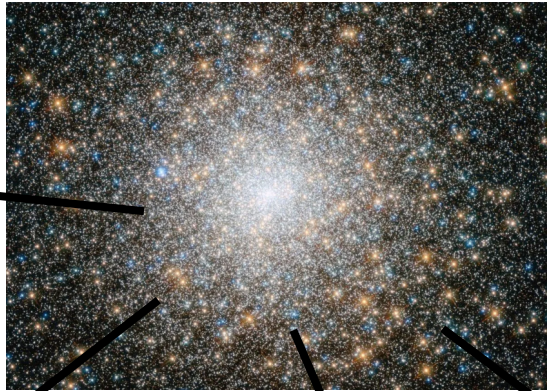
Astrophysical environments and scenarios

Star (globular/nuclear) cluster

AGN disk

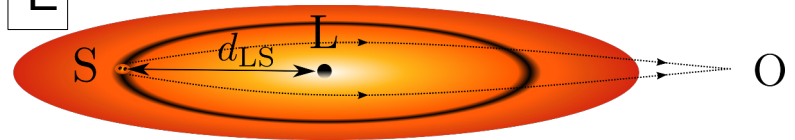
General optical depth for the detectable BH population

A



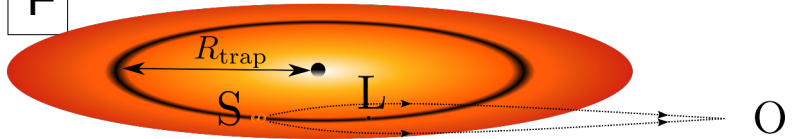
E

E. Central self-lensing

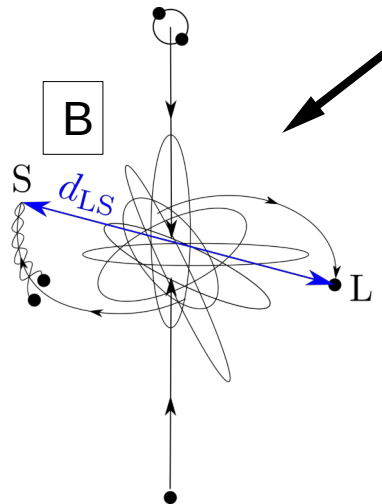


F

F. Lateral self-lensing

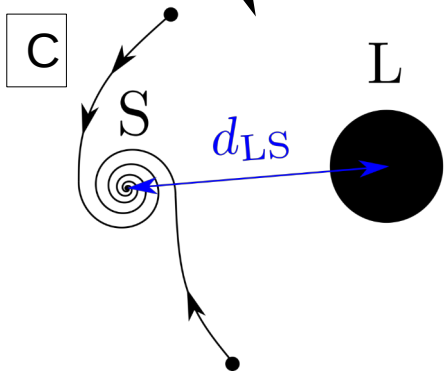


B



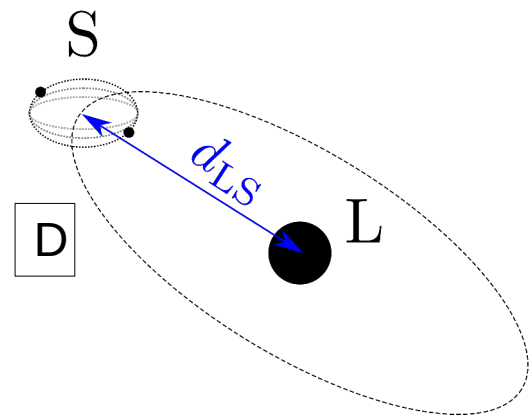
Resonant interactions

C



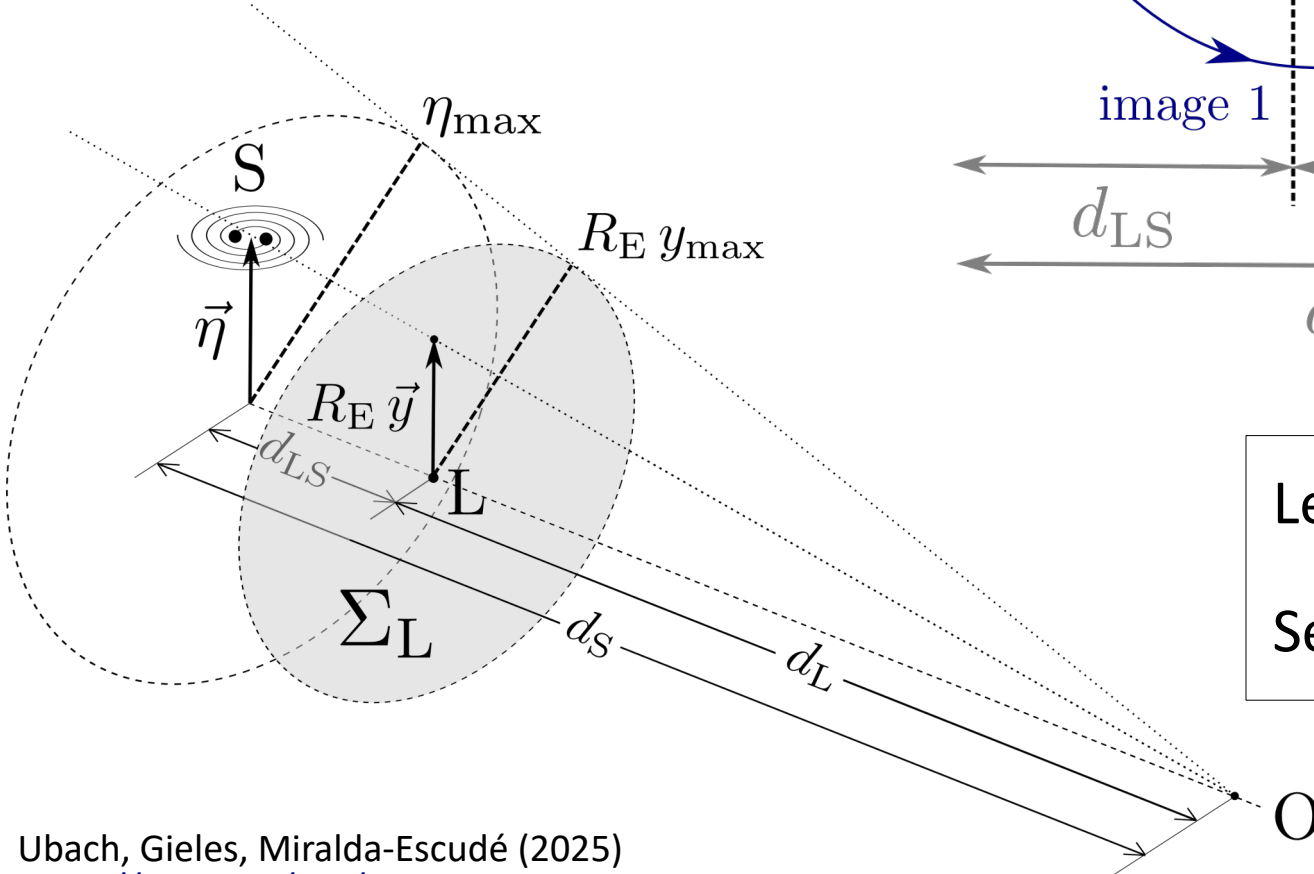
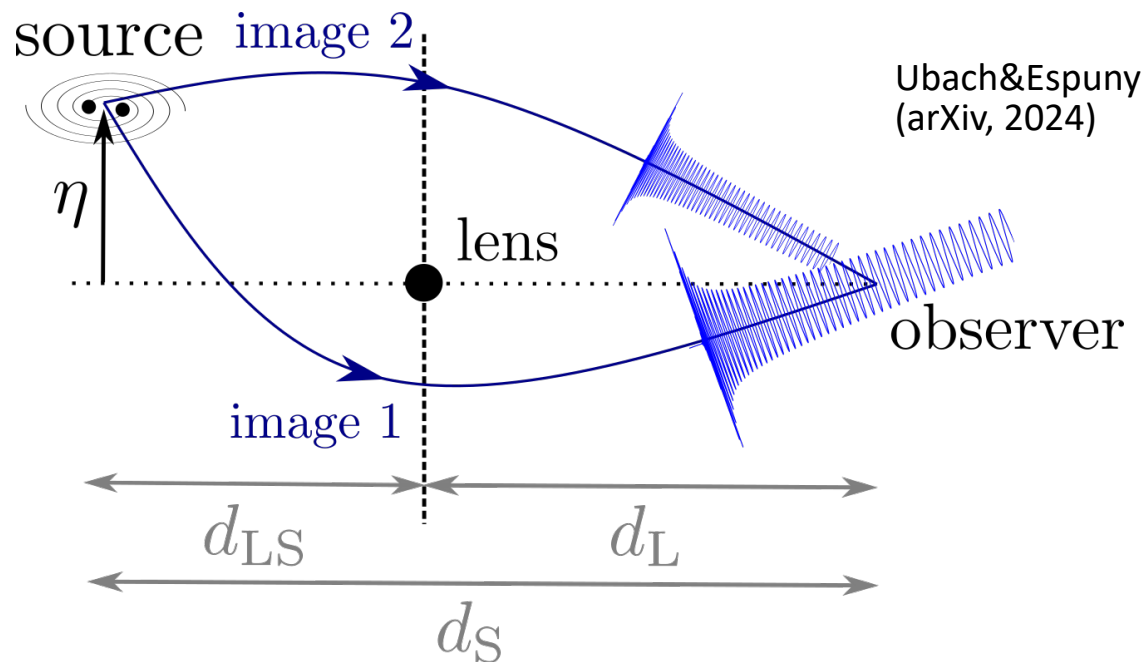
GW capture near central MBH

D



von Zeipel-Lidov-Kozai oscillations

5. Self-lensing



Lens model: point mass
Self-lensing: $d_{LS} \ll d_L, d_S$

Probability of self-lensing

- Probability $\simeq$ optical depth τ

$$\tau \propto \frac{\sigma^2}{c^2}$$

population of lenses,
optical lensing

$$\tau \propto \frac{v_{\text{orbit}}^2}{c^2} \propto \frac{R_L}{d_{\text{LS}}} \overset{\nearrow}{\propto} \frac{2Gm_L}{c^2}$$

one lens, e.g. Gould (1995)

→ low value: self-lensing usually neglected

Nevertheless,

- very dense (relativistic) environments, $\sigma \uparrow$
- # detections $\uparrow\uparrow$ (next generation detectors)
 - low probability $\neq$ impossible

5. Self-lensing

Probability of **detectable** lensing

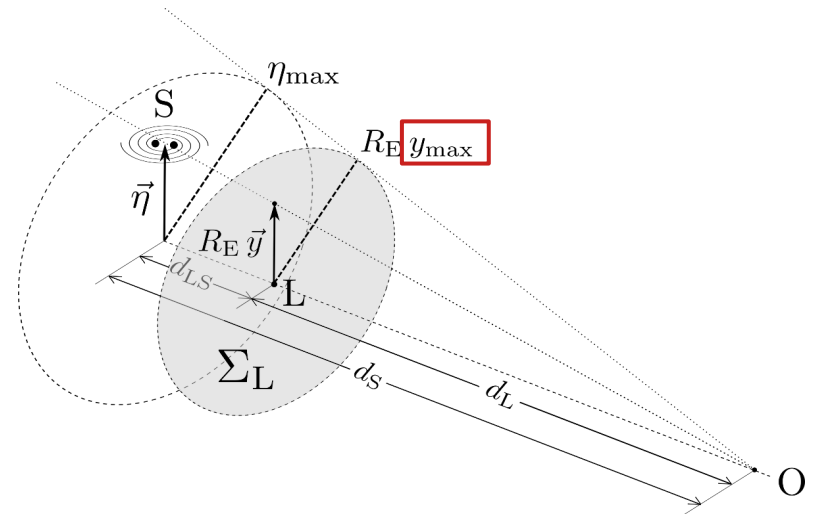
Detectability:

- Distortion (diffraction, interference)
- Multiple images

$$\tau \propto \frac{\sigma^2}{c^2} \boxed{y_{\max}^2}$$

$$\tau \propto \frac{v_{\text{orbit}}^2}{c^2} \boxed{y_{\max}^2} \propto \frac{R_L}{d_{LS}} \boxed{y_{\max}^2}$$

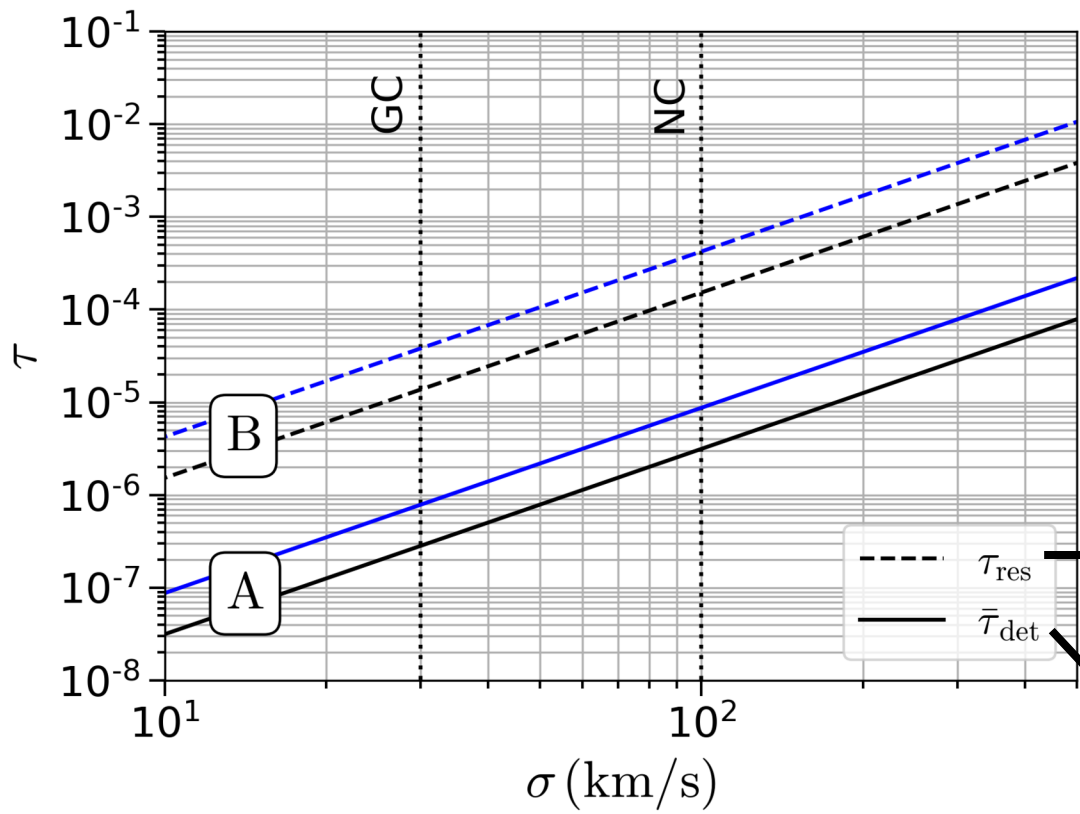
+ considering
just detectable
lens population



5. Self-lensing

Probability of self-lensing

- Probability of self-lensing (population): $\tau \propto \frac{\sigma^2}{c^2} y_{\max}^2$

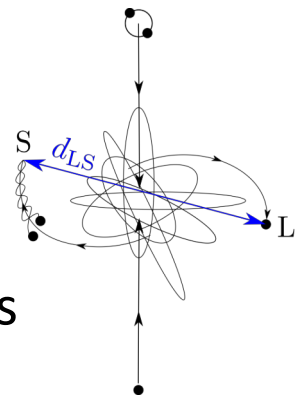


$y_{\max} \uparrow$ when SNR $\uparrow$

$y_{\max} = 2.5$
 $y_{\max} = 1.5$

B

Resonant interactions



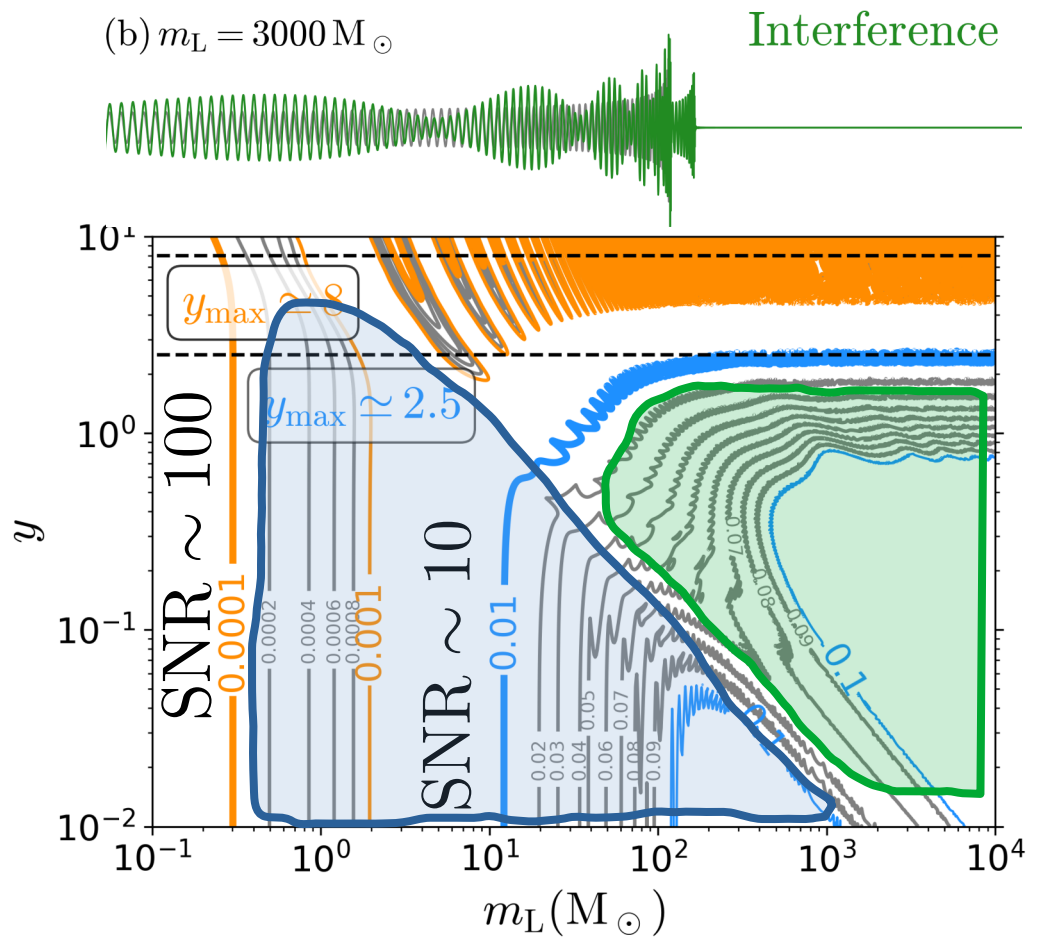
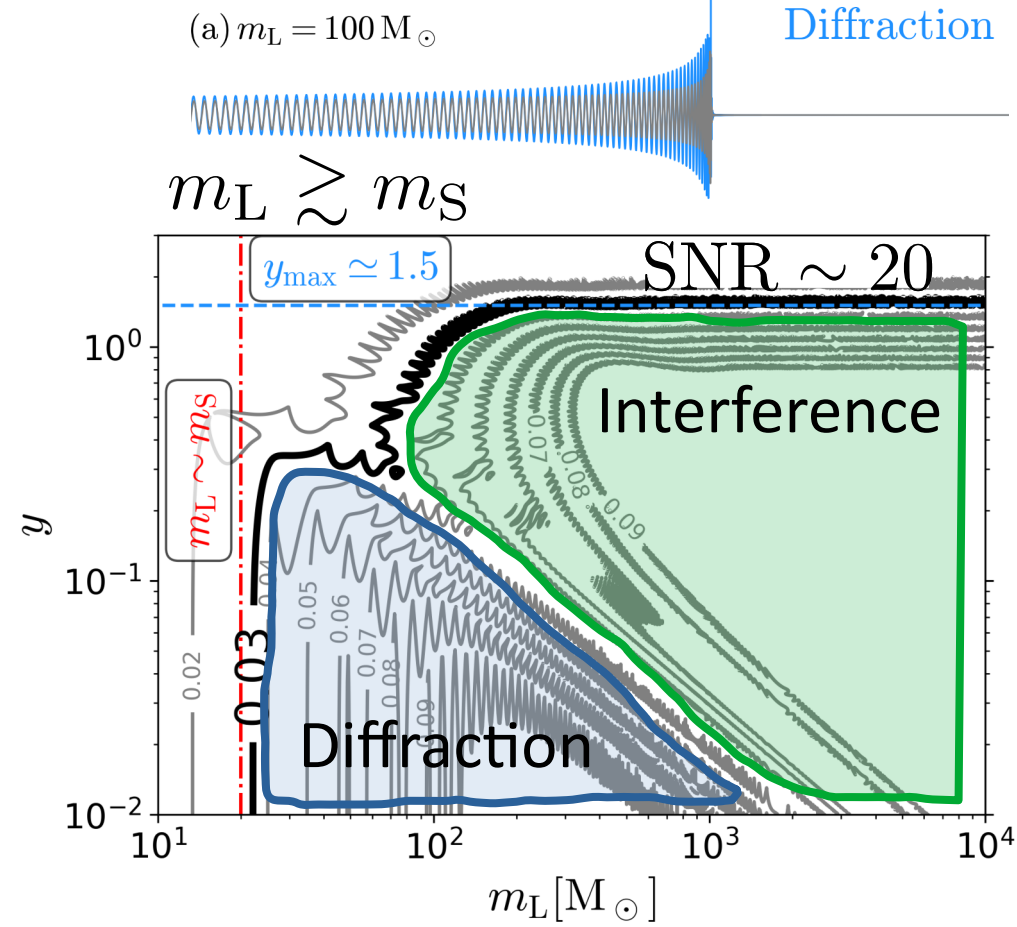
A

General optical depth for the detectable BH population

5. Self-lensing

Detectability

- Distortion on single waveform (diffraction, interference)



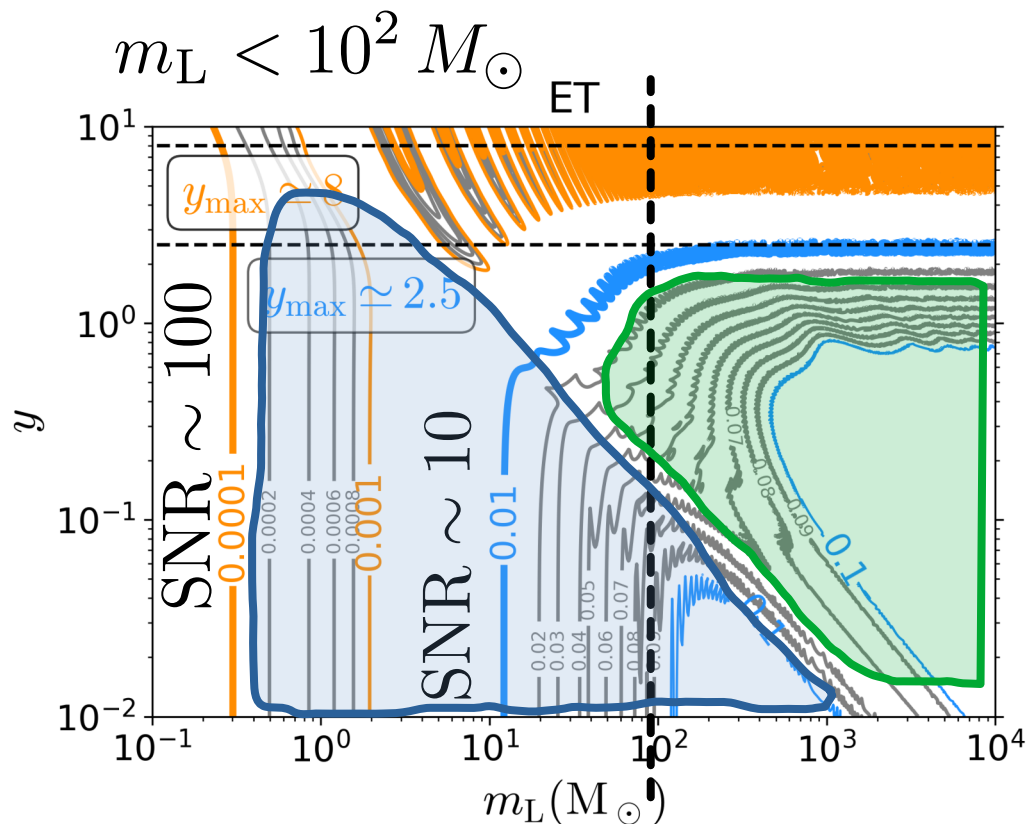
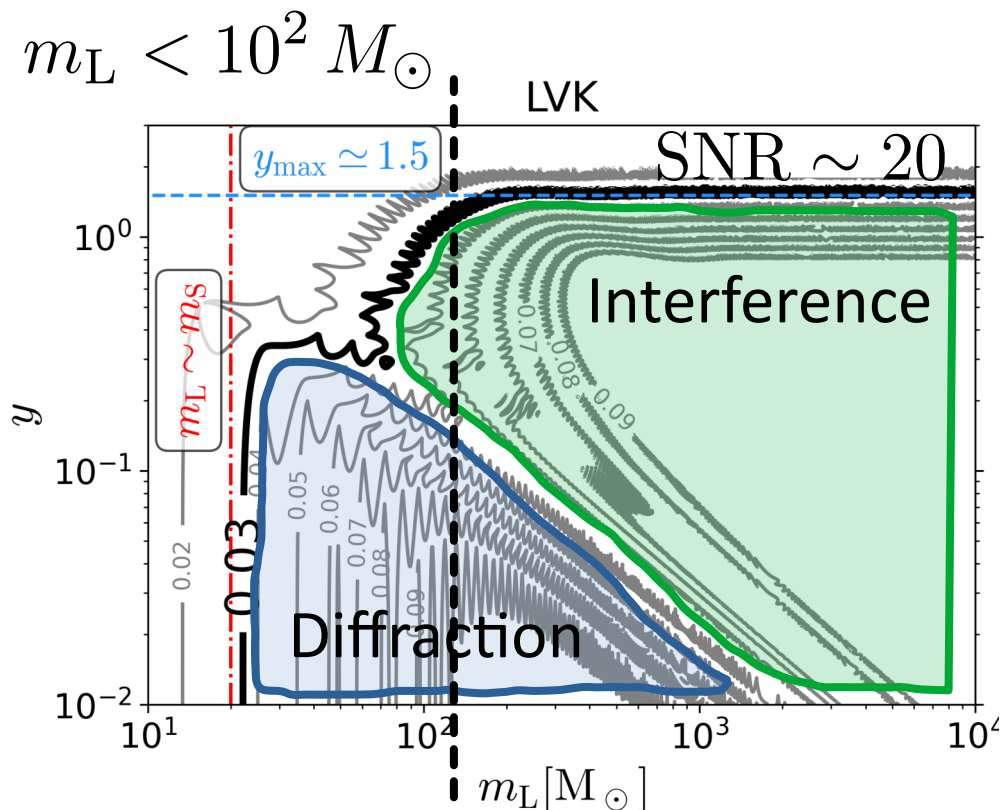
$$\text{Mismatch} \times \text{SNR}^2 > \ln B \sim (1 - 10)$$

Mismatch = $1 - \mathcal{M}$ Lindblom+(2008), Cornish+ (2011) Gao+ (2022), Tambalo+(2023), Savastano+(2023)

5. Self-lensing

Detectability

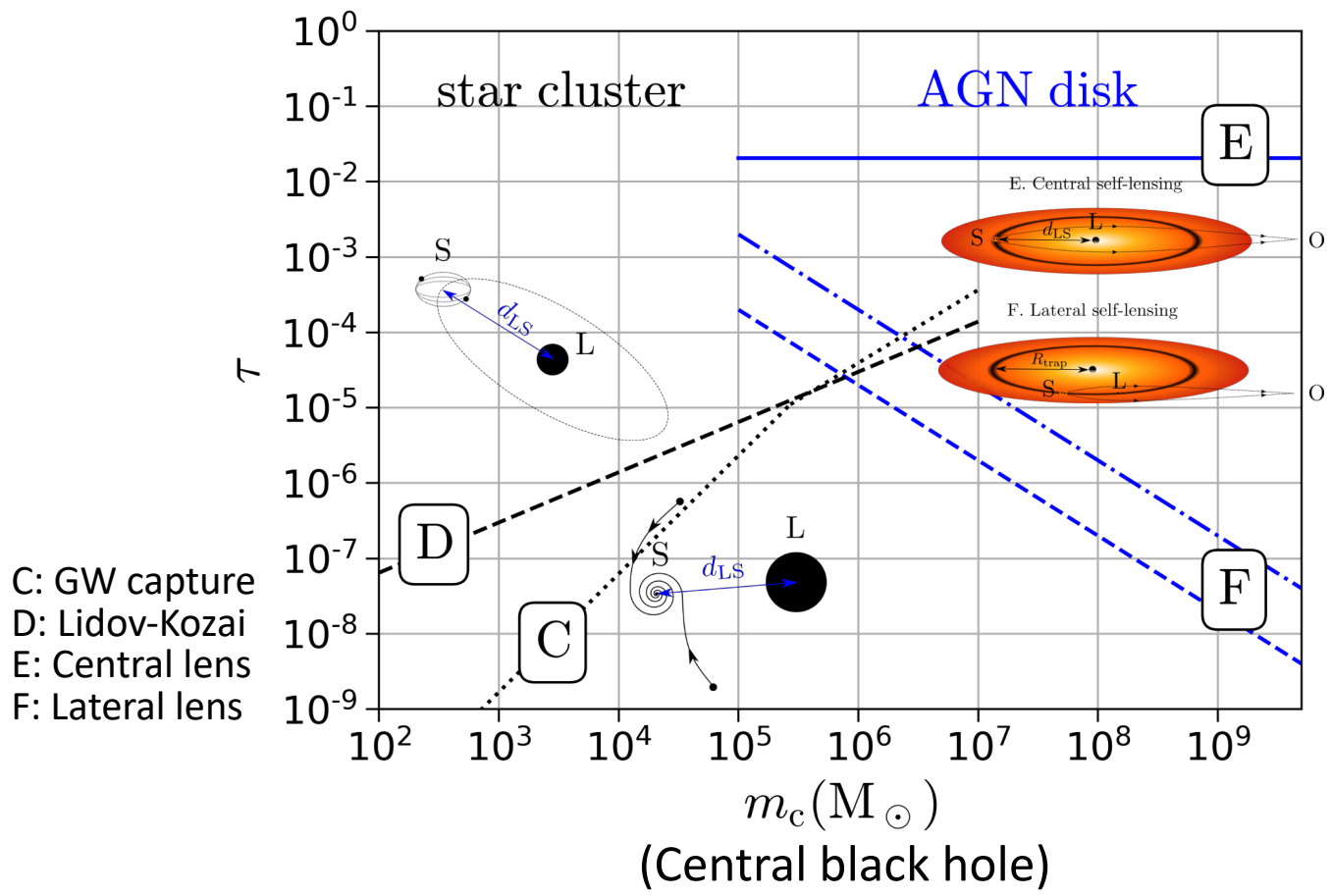
Stellar-mass BH lenses: mostly undetectable, $m_L \sim m_S$,



→ To have detectable imprint (interference pattern / multiple images), we need a massive BH in the star cluster (IMBH/SMBH)

5. Self-lensing

Probability



← Coincident with
Gondán&Kocsis (2022),
Leong et al. (2024)

Self-lensing probability
by central BH:

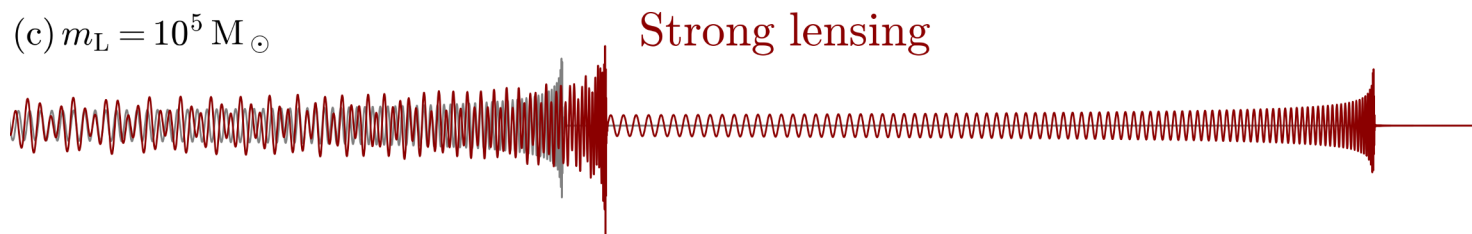
$$\tau \propto \frac{v_{\text{orbit}}^2}{c^2} \propto \frac{M_{\text{Lens}}}{d_{LS}}$$

Higher probability when
source (binary) is closer
to lens (central massive
black hole), $d_{LS} \downarrow$

Detectability

- Multiple images (strong lensing), $m_L \gtrsim 10^3 M_\odot$

(c) $m_L = 10^5 M_\odot$



$\text{SNR} > 8$ for both images

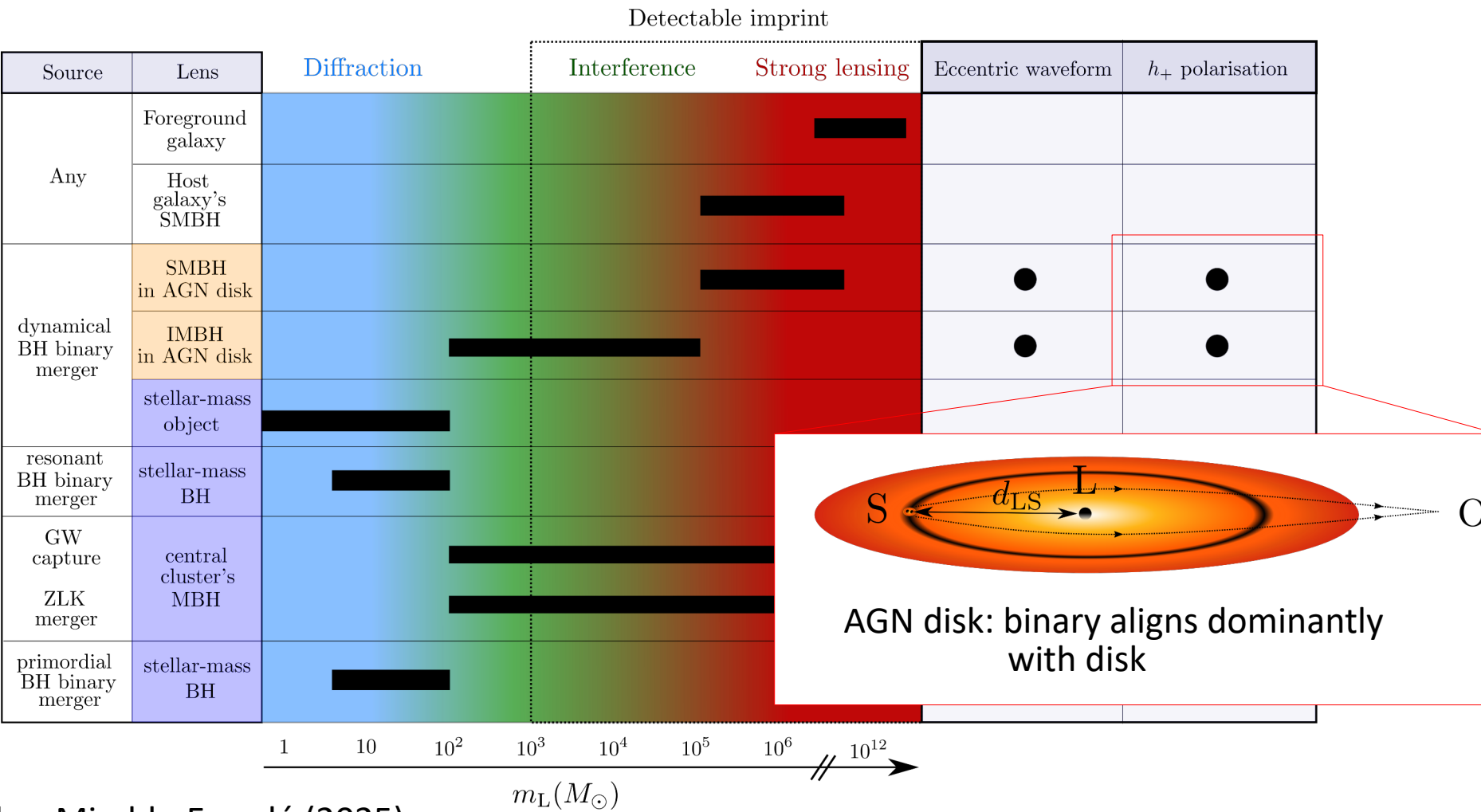
$$\left(\frac{\mu_+}{\mu_-} \right) = \left(\frac{\text{SNR}_{\text{lensed}}^+}{\text{SNR}_{\text{lensed}}^-} \right)^2 \rightarrow y_{\text{max}}$$

For LVK, $(\text{SNR}_L^+)^{\text{max}} = 50$, $y_{\text{max}} \simeq 2$.

For ET, $(\text{SNR}_L^+)^{\text{max}} = 1000$, $y_{\text{max}} \simeq 10$.

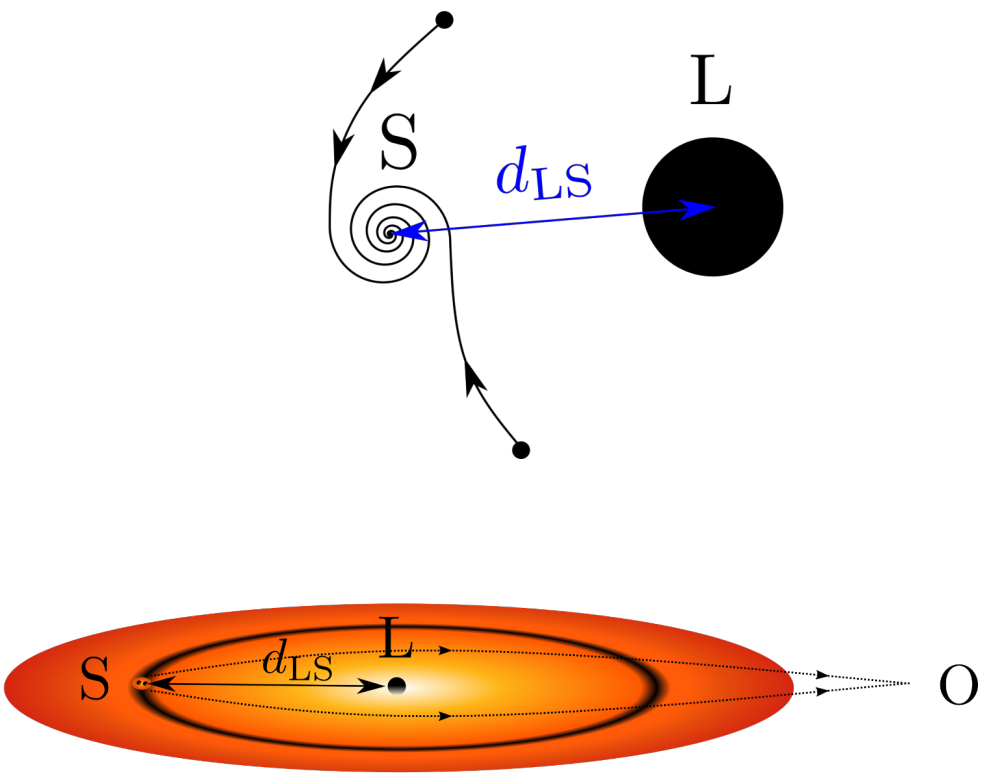
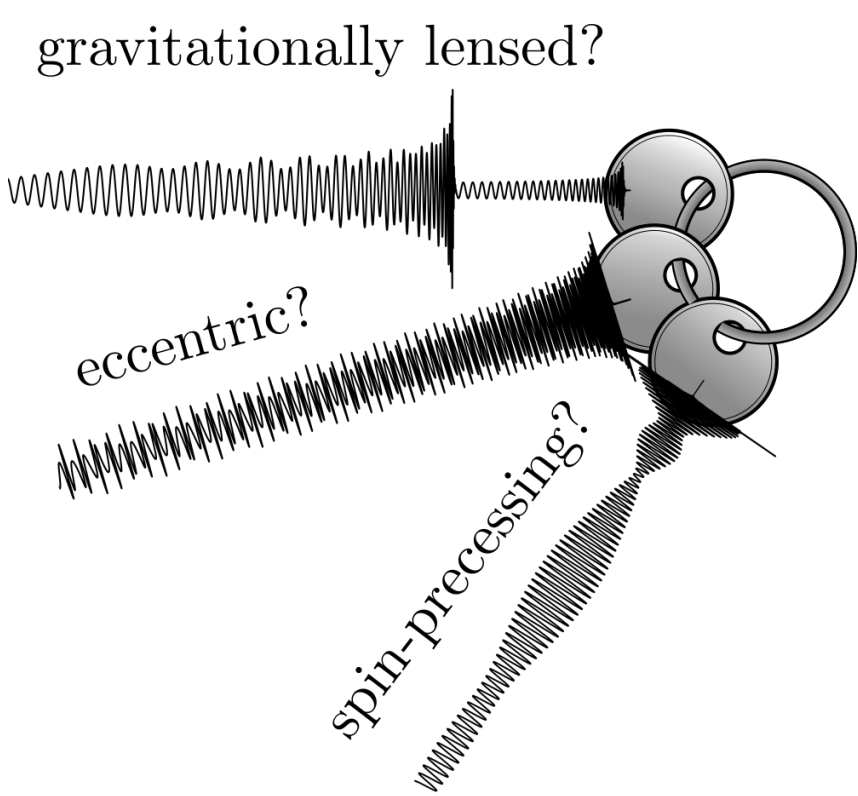
5. Self-lensing

Signatures to distinguish between astrophysical environments



5. Self-lensing

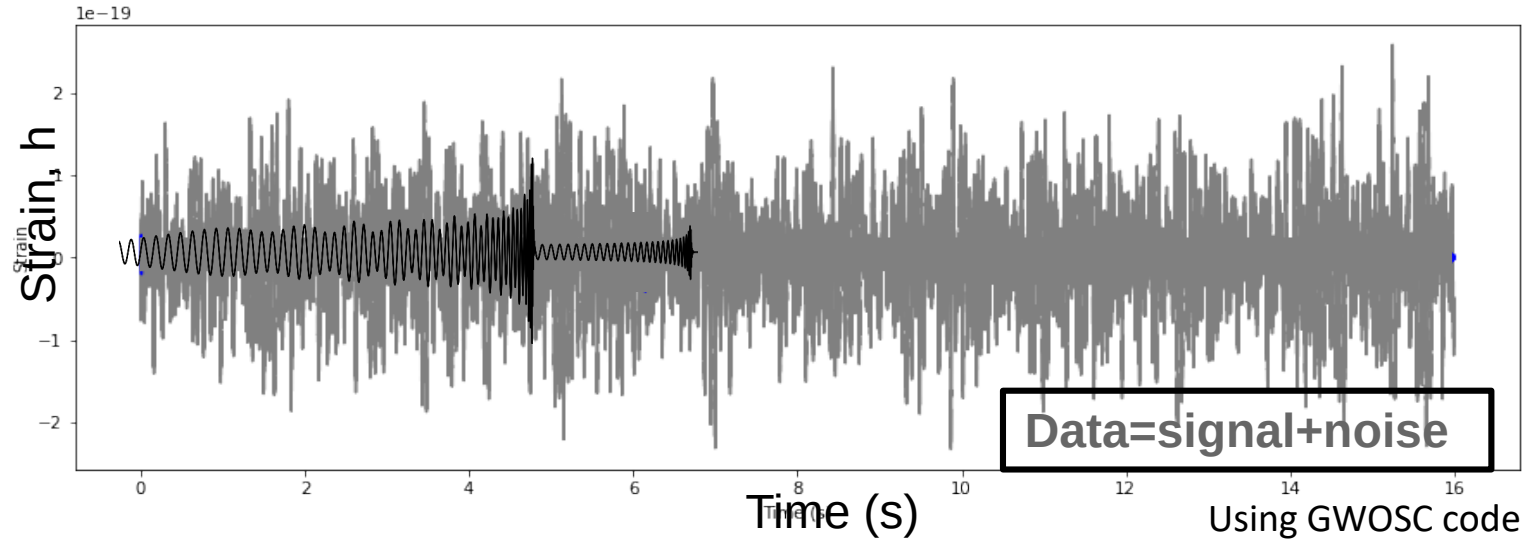
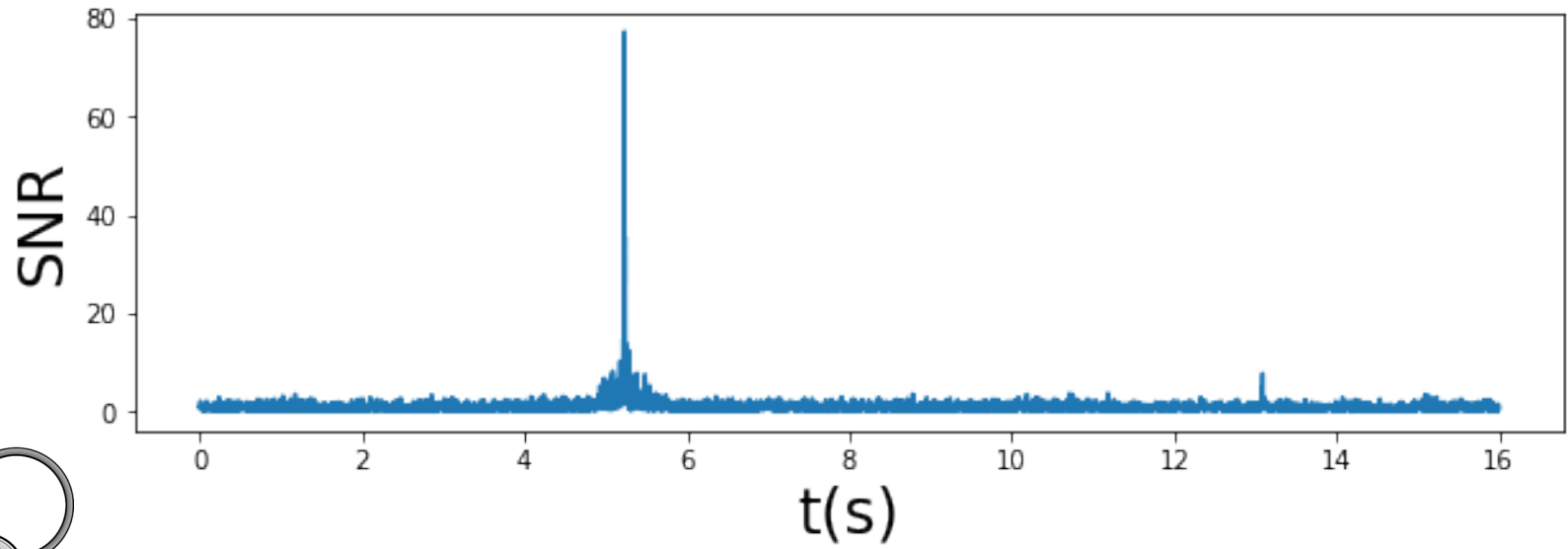
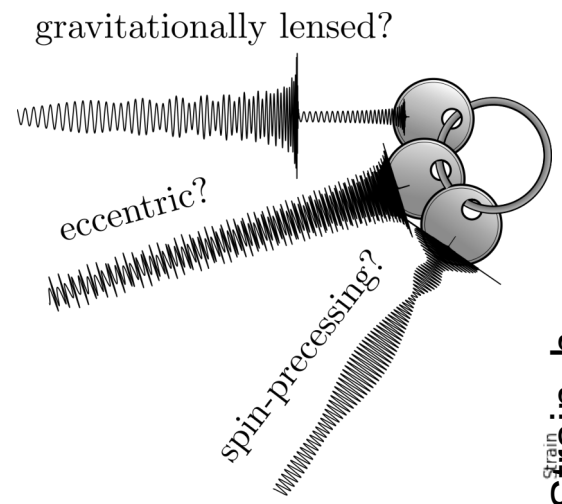
The keys to dynamical formation environments



Waveforms obtained using pycbc

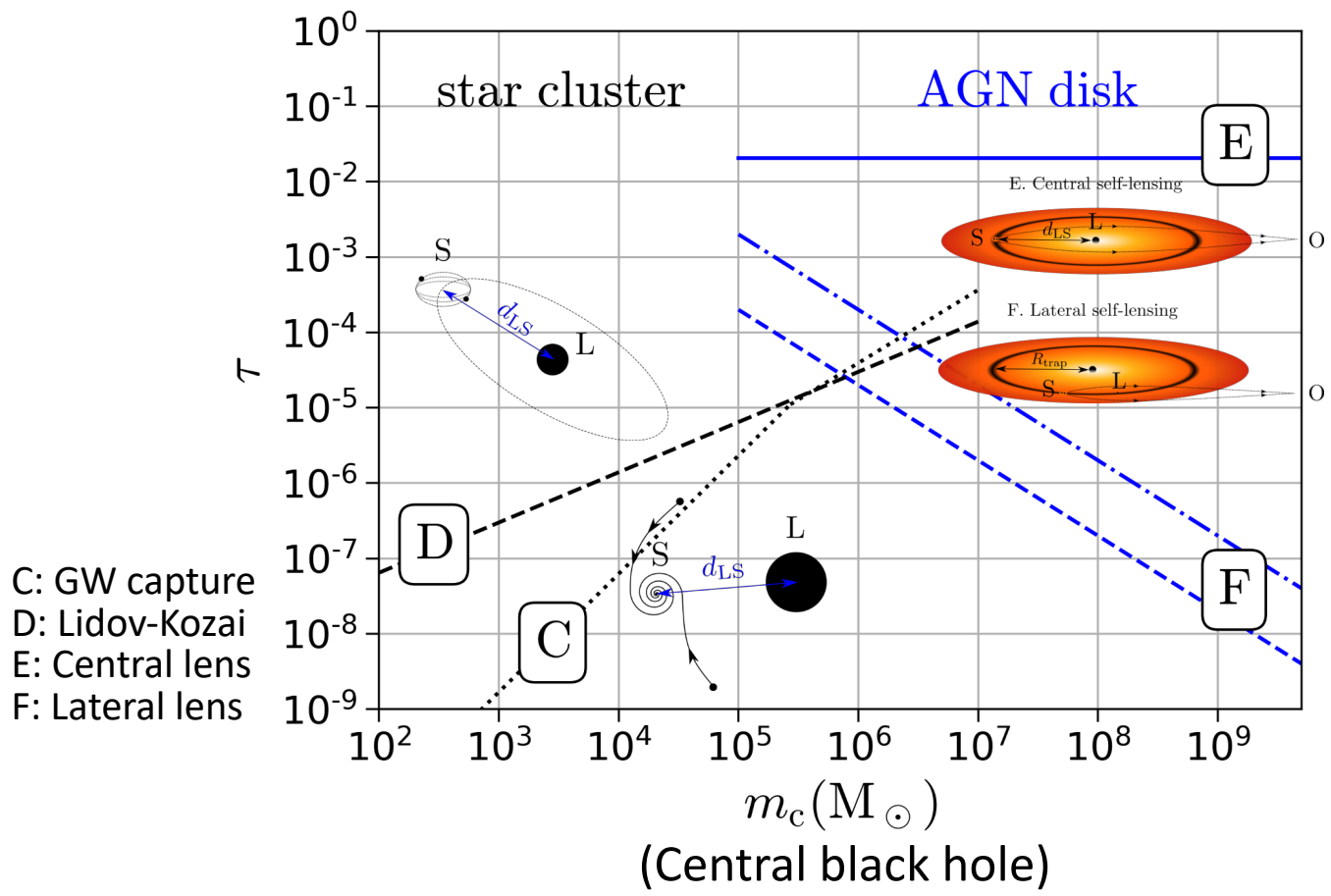
5. Self-lensing

The keys to dynamical formation environments



5. Self-lensing

Probability



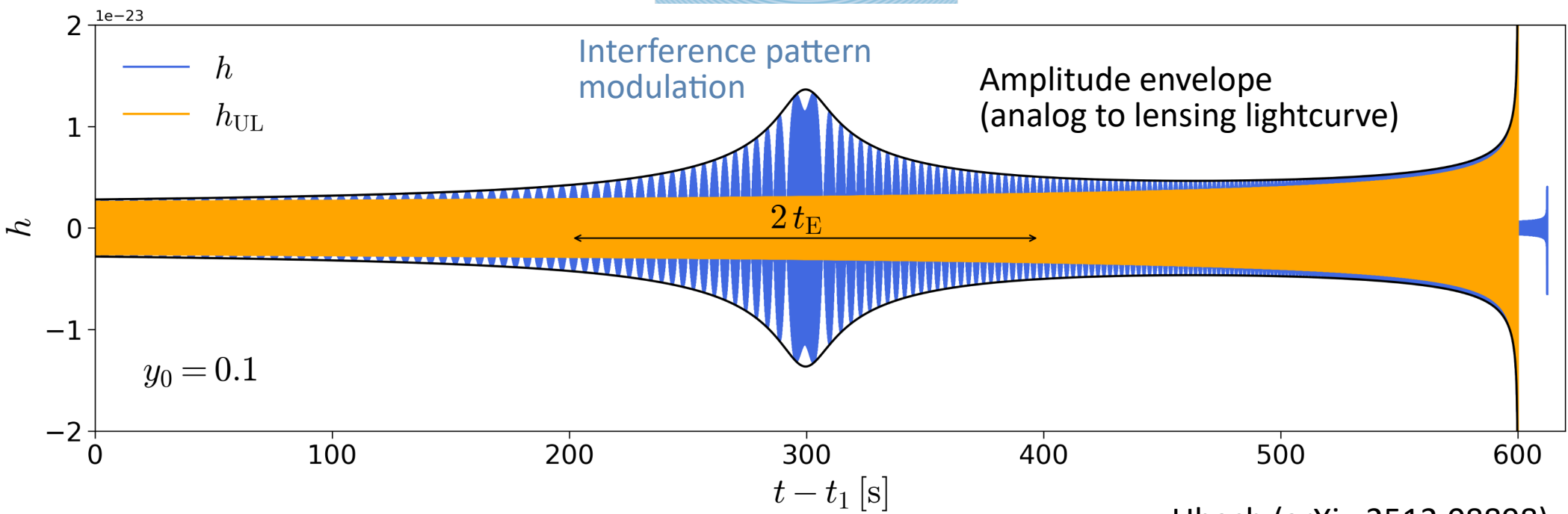
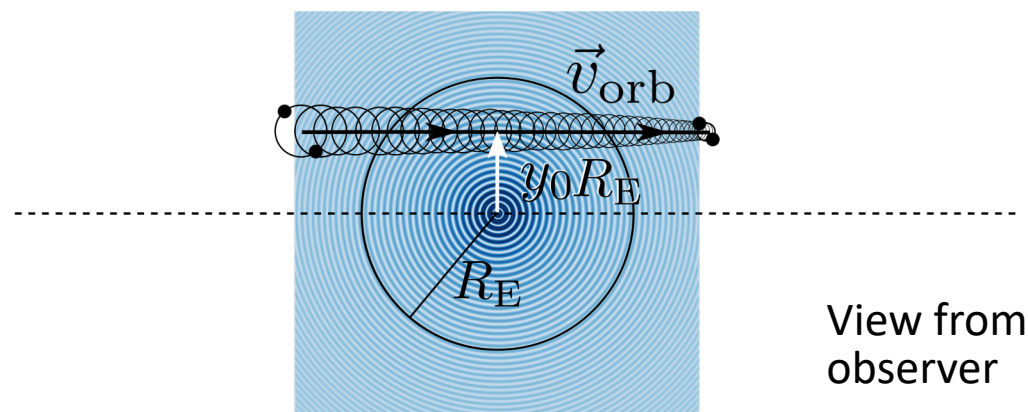
← Coincident with
Gondán&Kocsis (2022),
Leong et al. (2024)

Self-lensing probability
by central BH:

$$\tau \propto \frac{v_{\text{orbit}}^2}{c^2} \propto \frac{M_{\text{Lens}}}{d_{\text{LS}}}$$

Higher probability when
source (binary) is closer
to lens (central massive
black hole), $d_{\text{LS}} \downarrow$

6. Self-lensing of a moving source



6. Self-lensing of a moving source

Breaking a microlensing degeneracy:
obtaining the **orbital distance**

Obtain information from t_E (half-width of the curve)? **Usually degenerate parameters**

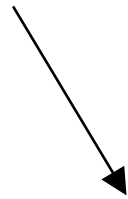
$$t_E = \frac{1}{v} \sqrt{\frac{4GM}{c^2} \frac{d_L d_{LS}}{d_S}}$$

When the source orbits the lens,

$d_L \approx d_S$

circular orbit

$v = v_{\text{orb}} \simeq \sqrt{\frac{GM}{d_{LS}}}$


$$t_E = \frac{2d_{LS}}{c}$$

Measuring t_E ,
we can directly know the
orbital distance d_{LS}

But what about M ?

6. Self-lensing of a moving source

Obtaining the **(redshifted) lens mass**

The modulations (interference pattern) depend on M . They are created by the interference between the two signals:

$$\tilde{h}(f) = \tilde{h}_{\text{UL}}(f) e^{i2\pi f t_1} \left[\sqrt{\mu_1} + \sqrt{\mu_2} e^{i2\pi f \Delta t - i\pi/2} \right]$$

Time delay is changing over time as the source moves

$$\Delta t \simeq 4y(t) R_S (1+z)/c \quad \text{where } y(t) = \sqrt{y_0^2 + (t - t_0)^2/t_E^2}$$

Modulation period comes from phase difference between images: $e^{i2\pi f \Delta t} \equiv e^{i2\pi t/T}$

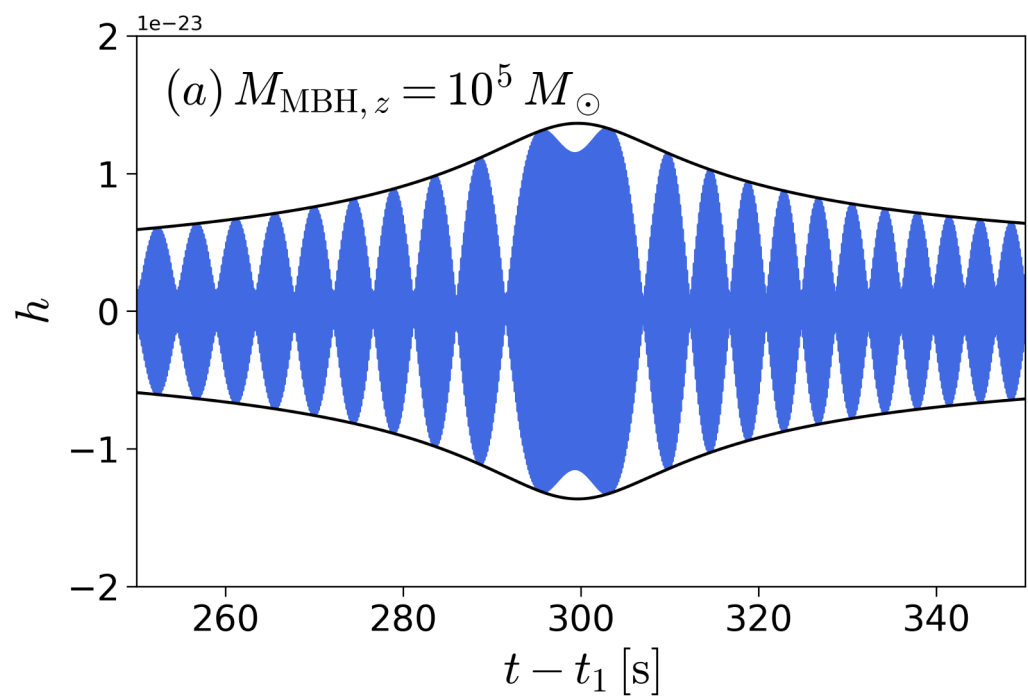
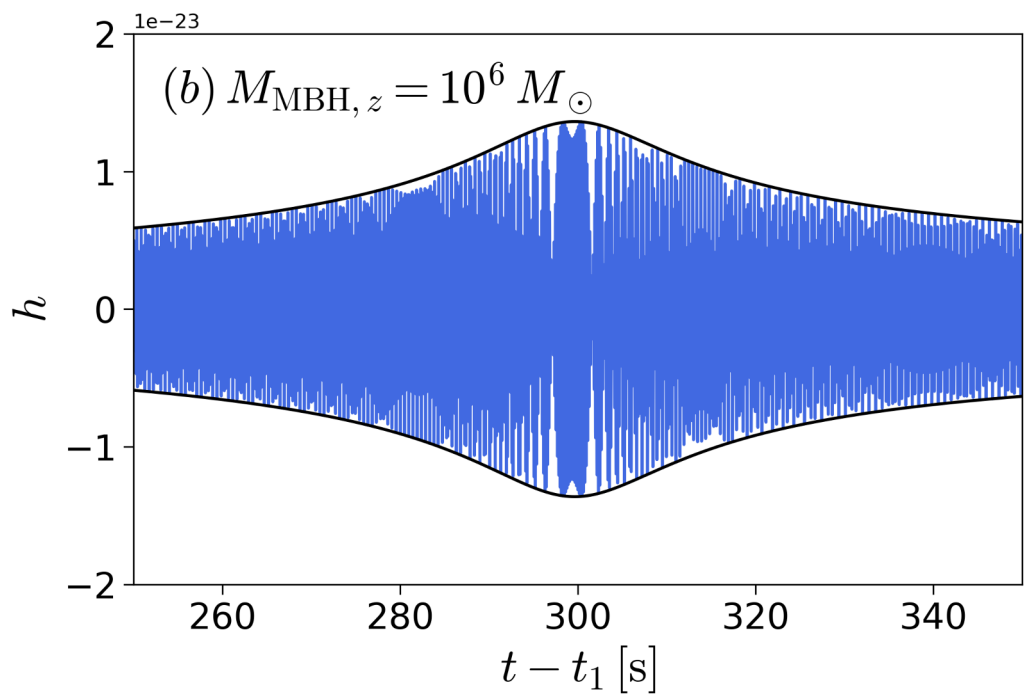
$$T = \frac{t}{f \Delta t} \propto \frac{1}{M_{\text{Lens},z}} \quad \text{where } M_{\text{Lens},z} = M(1+z)$$

$$M_{\text{Lens},z} \simeq 2.5 \times 10^6 M_\odot (t_E/[100 \text{ s}]) (f T)^{-1}$$

6. Self-lensing of a moving source

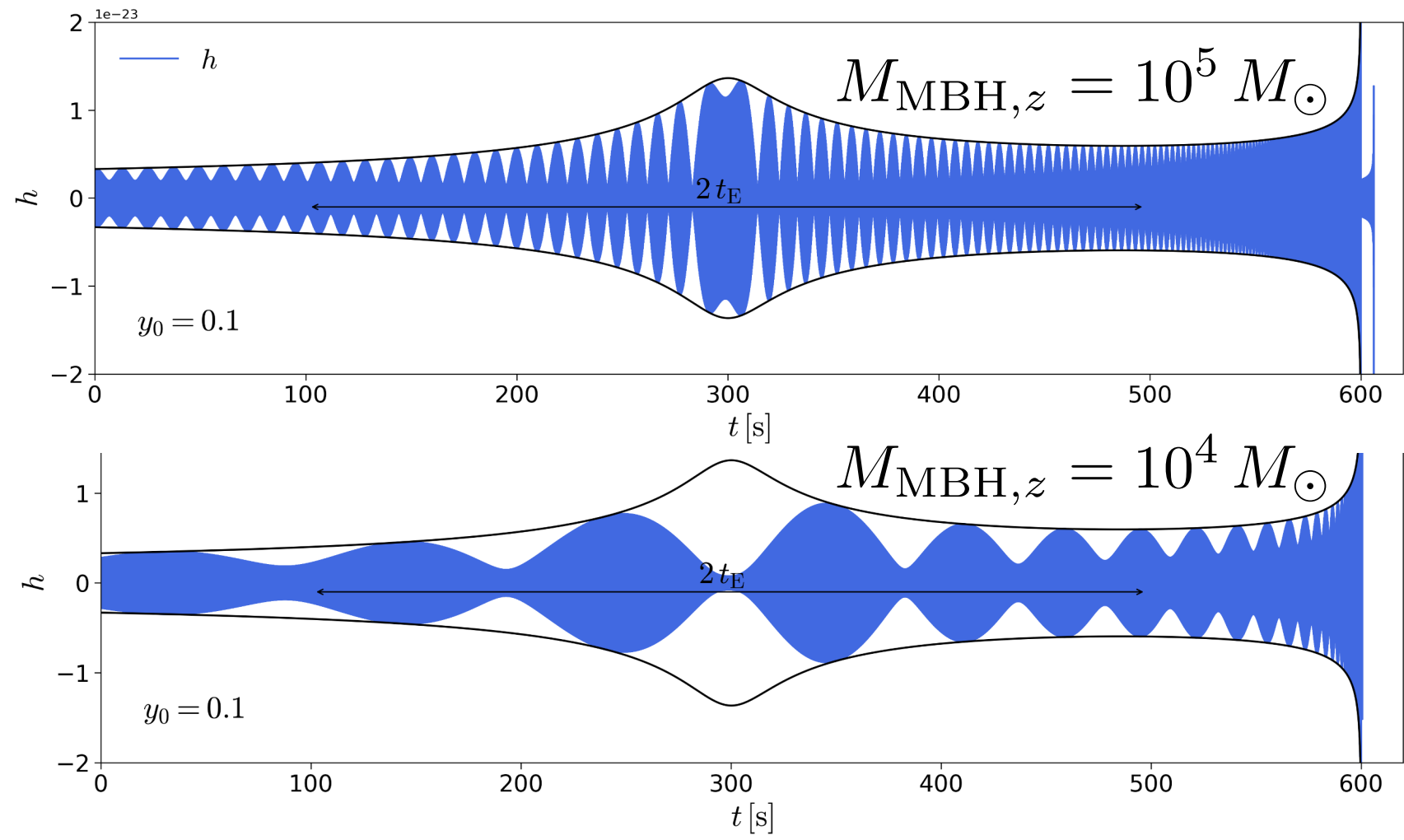
Dependence on the (redshifted) lens mass

$$M_{\text{MBH},z} \simeq 2.5 \times 10^6 M_{\odot} (t_{\text{E}}/[100 \text{ s}]) (f T)^{-1}$$



6. Self-lensing of a moving source

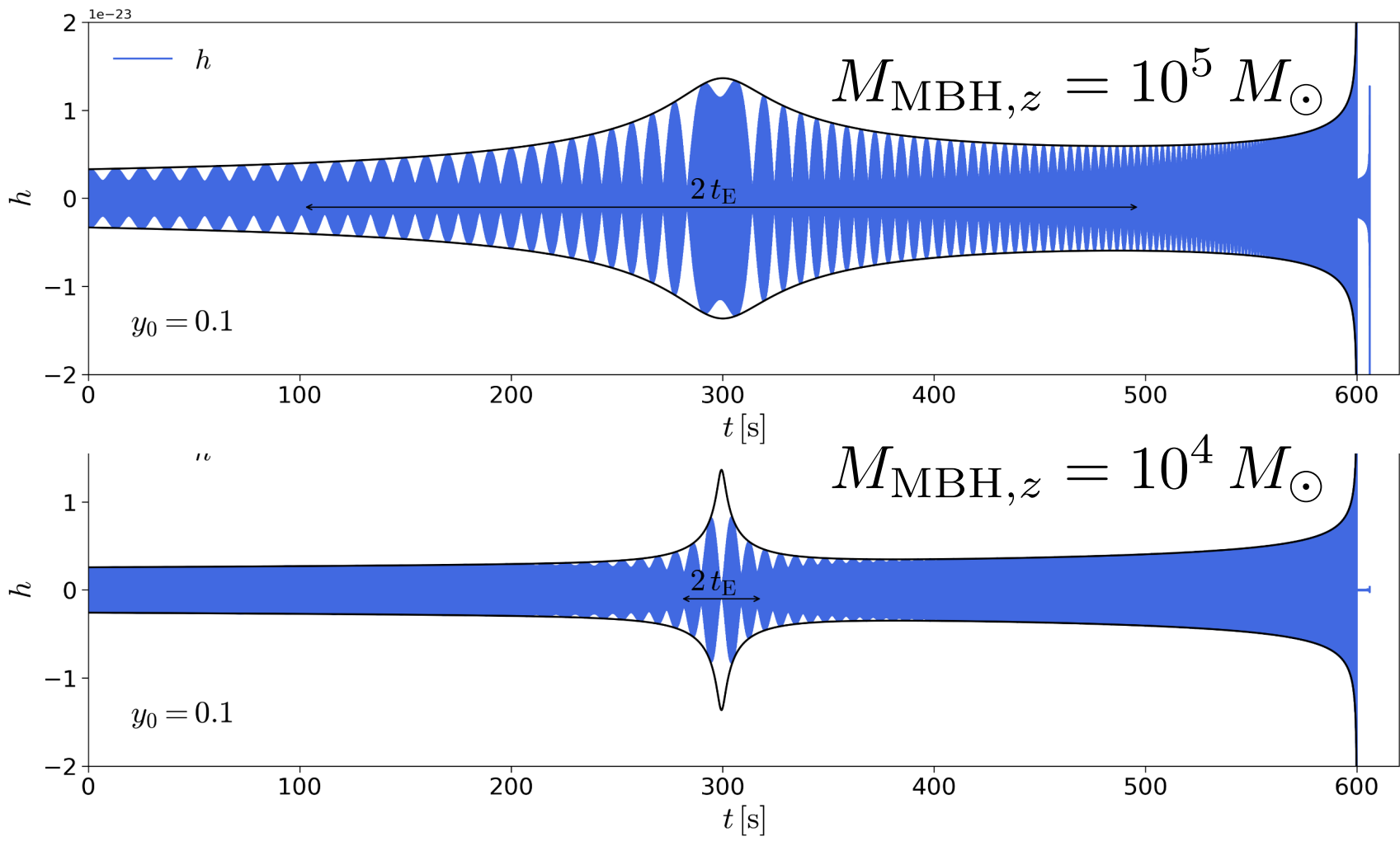
$$M_{\text{MBH},z} \simeq 2.5 \times 10^6 M_{\odot} (t_{\text{E}}/[100 \text{ s}]) (f T)^{-1}$$



$$d_{\text{LS}} = 0.2 \text{ AU}$$

6. Self-lensing of a moving source

$$M_{\text{MBH},z} \simeq 2.5 \times 10^6 \, M_\odot \, (t_{\text{E}}/[100 \, \text{s}]) \, (f \, T)^{-1}$$



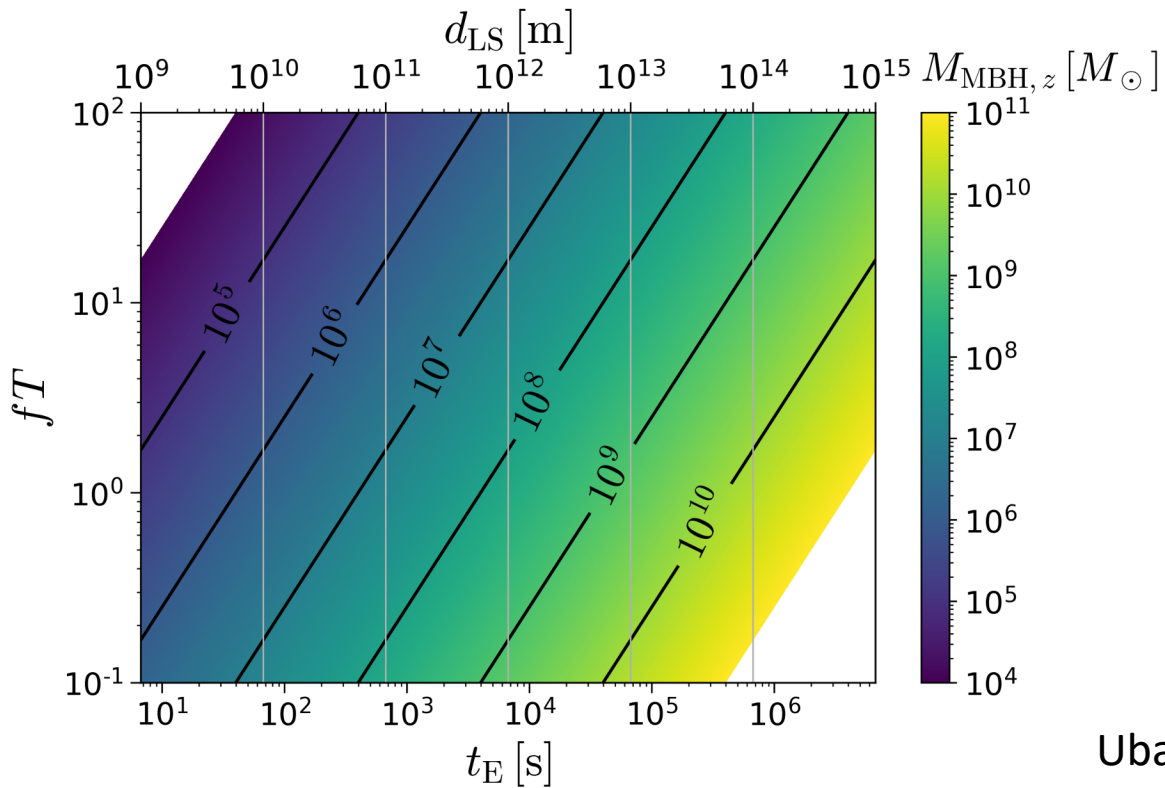
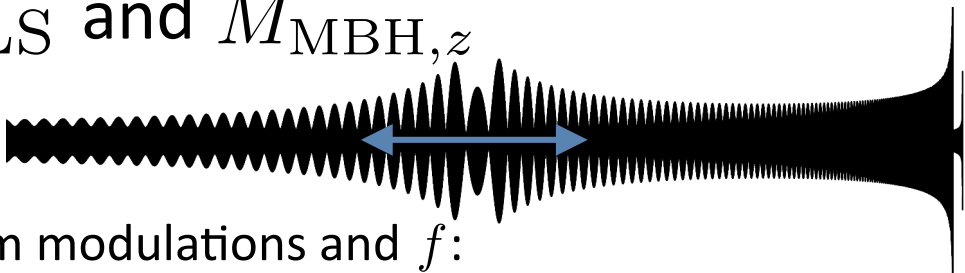
$$d_{\text{LS}} = 100 \, R_{\text{S}} \approx 2 \, \text{AU} \, (M/[10^6 \, M_\odot])$$

6. Self-lensing of a moving source

Constraints on d_{LS} and $M_{\text{MBH},z}$

- d_{LS} from measuring t_{E} : $d_{\text{LS}} = c t_{\text{E}}/2$
- $M_{\text{MBH},z}$ from additionally measuring T from modulations and f :

$$M_{\text{MBH},z} \simeq 2.5 \times 10^6 M_{\odot} (t_{\text{E}}/[100 \text{ s}]) (f T)^{-1}$$



Outlook

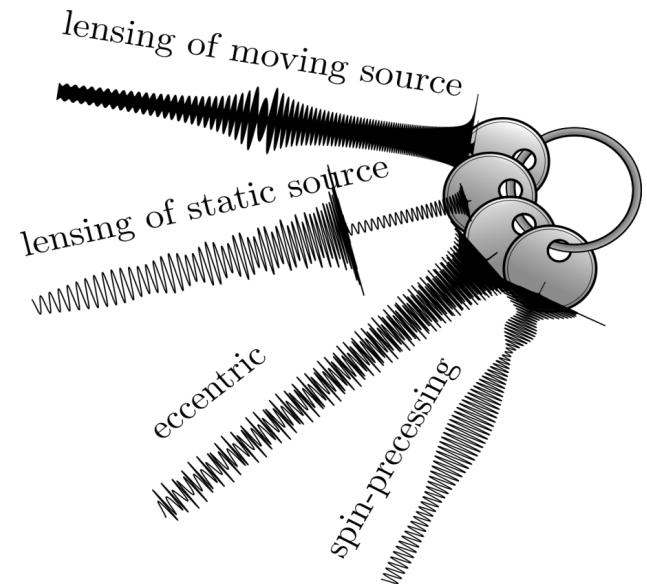
Wave effects can probe intermediate-mass black holes regime

Interference is potentially detectable,
diffraction mostly undetectable for the moment

Self-lensing can provide additional information
to constrain astrophysical environment

New lensing feature: moving source

- Characteristic of dense environments (high velocity)
- Additional information from interference pattern
- Next steps: detectability and degeneracies



Funding

Grants

- PID2024-159689NB-C22
- CEX2024-001451-M funded by MCIN/AEI/10.13039/501100011033 (State Agency for Research of the Spanish Ministry of Science and Innovation)
- SGR-2021-01069 (AGAUR, Generalitat de Catalunya)

Fellowship

- FI-SDUR 2023 predoctoral grant (AGAUR, Generalitat de Catalunya)

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Extra slides

6. Self-lensing of a moving source

Dependence on the minimum frequency

